

Sensitivity Analysis Using SensIt

4

4.1 SENSIT OVERVIEW

SensIt is a sensitivity analysis add-in for Microsoft Excel 2000–2010 (Windows) and Microsoft Excel 2004 & 2011 (Macintosh).

SensIt performs sensitivity analysis on your worksheet what-if model and creates simple plots, spider charts, and tornado charts. Sensitivity analysis can help you determine which of your input assumptions are critical.

Before using the SensIt options, you must have a spreadsheet model with one or more inputs and an output. With SensIt it is easy to see how sensitive the output is to changes in the inputs. In Excel 2007 or 2010, SensIt adds a Sensitivity Analysis command to the Add-Ins ribbon. In Excel 2003 and earlier, SensIt adds a Sensitivity Analysis command to the Tools menu. This command has three subcommands: One Input, One Output; Many Inputs, One Output; and Help.

Use SensIt's One Input, One Output option to see how your model's output depends on changes in a single input variable. This feature creates an XY (Scatter) chart type.

Use SensIt's Many Inputs, One Output option to see how your model's output depends on ranges you specify for each of the model's input variables. This feature creates tornado charts (an Excel Bar chart type) and a spider chart (an XY (Scatter) chart type). The spider chart is useful for detecting a nonlinear relationship between the input variables and the output variable of your model. The two-factor tornado chart can help you determine how your model output is affected by each pair of your input assumptions.

All of SensIt's functionality, including its built-in help, is a part of the SensIt XLA file. There is no separate setup file or help file. When you use SensIt on a Windows computer, it does not create any Windows Registry entries (although Excel may use such entries to keep track of its add-ins).

4.2 EXAMPLE PROBLEM

This example is a simple economic model of a prospective new product taken from the Eschenbach (1992) article. The model assumes that the FirstCost investment occurs at the start of the first year and all subsequent cash flows are end-of-year. The product life is N years with the Salvage value at the end of year N . At the start of the project there is no revenue for the first N_{NoRev} years with subsequent annual Revenue during years $N_{NoRev}+1$ through N . $FracComp$ is the fraction of Revenue lost to competition. Annual operations and maintenance cost $O\&M$ applies for all N years of product life. The discount rate i is used to determine PresentWorth. Eschenbach uses the notation $P/A,i,N$ and $P/F,i,N$, respectively, for the uniform periodic and the

single payment present worth factors. The spreadsheet model includes some intermediate variables for determining various discount factors.

Figure 4.1 Model Display

	A	B
1	Input Variables	Input Cells
2	FirstCost	\$120,000
3	Salvage	\$20,000
4	N	12
5	i	10%
6	O&M	\$6,000
7	Revenue	\$55,000
8	NNoRev	1
9	FracComp	0.2
10	Intermediate Variables	
11	P/F,i,N	\$0.32
12	P/A,i,N	\$6.81
13	N-NNoRev	11
14	P/A,i,N-NNoRev	\$6.50
15	P/F,i,NNoRev	\$0.91
16	Revenue Discount	\$5.90
17	Output Variable	
18	Present Worth	\$105,293

Figure 4.2 Model Formulas

	A	B
10	Intermediate Variables	
11	P/F,i,N	=1/(1+B5)^B4
12	P/A,i,N	=PV(B5,B4,-1)
13	N-NNoRev	=B4-B8
14	P/A,i,N-NNoRev	=PV(B5,B13,-1)
15	P/F,i,NNoRev	=1/(1+B5)^B8
16	Revenue Discount	=B14*B15
17	Output Variable	
18	Present Worth	=-B2+B3*B11-B6*B12+B7*(1-B9)*B16

4.3 ONE INPUT, ONE OUTPUT

Use SensIt's One Input, One Output option to see how your model's output depends on changes in a single input variable. In Excel 2003, choose Tools | Sensitivity Analysis | One Input, One Output. In Excel 2007 or 2010, choose Add-ins | Sensitivity Analysis | One Input, One Output.

Figure 4.3 SensIt One Input, One Output Dialog Box

SensIt 1.45 - One Input, One Output

Cells for Input Variable

Label: OneInput!\$A\$4

Value: OneInput!\$B\$4

Cells for Output Variable

Label: OneInput!\$A\$18

Value: OneInput!\$B\$18

Input Values

Start: 6

Step: 1

Stop: 24

Reset All

OK

Cancel

Help

Cells for Input Variable

In the Label reference edit box, type a cell reference, or point to the cell containing a text label and click. In the Value reference edit box, type a cell reference, or point to the cell containing a numeric value that is an input cell of your model.

Cells for Output Variable

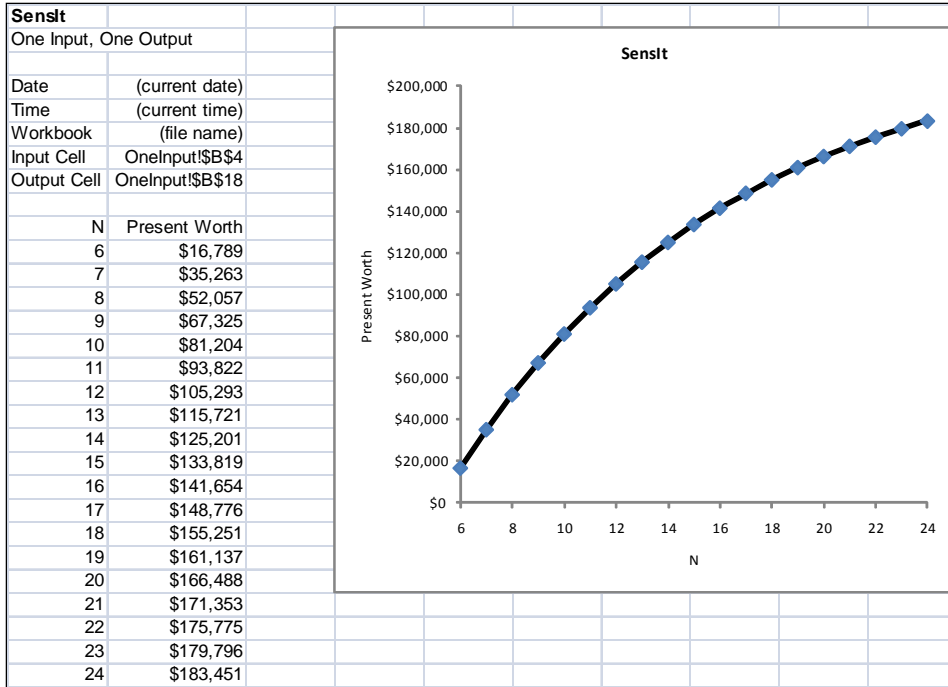
In the Label reference edit box, type a cell reference, or point to the cell containing a text label and click. In the Value reference edit box, type a cell reference, or point to the cell containing a formula that is the output of your model.

Input Values

Type numbers in the Start, Step, and Stop edit boxes to specify values to be used in the input variable's cell. Cell references are not allowed.

Click OK: SensIt uses the Start, Step, and Stop values to prepare a table of values. Each value is copied to the input variable Value cell, the worksheet is recalculated, and the value of the output variable Value cell is copied to the table. (You could do this manually in Excel using the Edit | Fill | Series and Data | Table commands.) SensIt uses the paired input and output values to prepare an XY (Scatter) chart. The text in the label cells you identified are used as the chart's axis labels. (You could do this manually using the ChartWizard.)

Figure 4.4 SensIt Numerical and Chart Output



Insight From the table and chart, we observe a nonlinear relationship between Present Worth and product life N. That is, the change in Present Worth between years 6 and 7 is much greater than the change between years 23 and 24.

4.4 MANY INPUTS, ONE OUTPUT TORNADO

Use SensIt’s Many Inputs, One Output option to see how your model’s output depends on ranges you specify for each of the model’s input variables. Before using this option, arrange your model input cells in adjacent cells in a single column, arrange corresponding labels in adjacent cells in a single column, and arrange Low, Base, and High input values for each input variable in three separate columns. Alternatively, the three columns containing input values can be worst case, likely case, and best case. An appropriate arrangement is shown below.

Figure 4.5 Model Display with Lower and Upper Bounds

	A	B	C	D	E	F	G	H	I
1	Input Variables	Input Cells		OneExtreme	BaseCase	OtherExtreme		LowerLimit%	UpperLimit%
2	FirstCost	\$120,000		\$108,000	\$120,000	\$180,000		90%	150%
3	Salvage	\$20,000		\$0	\$20,000	\$30,000		0%	150%
4	N	12		6	12	24		50%	200%
5	i	10%		6%	10%	20%		60%	200%
6	O&M	\$6,000		\$4,800	\$6,000	\$7,500		80%	125%
7	Revenue	\$55,000		\$33,000	\$55,000	\$68,750		60%	125%
8	NNoRev	1		0	1	3		0%	300%
9	FracComp	0.2		0	0.2	0.4		0%	200%
10	Intermediate Variables			(Input Values for SensIt)				(Eschenbach's Inputs Relative To Base Case)	
11	P/F,i,N	\$0.32							
12	P/A,i,N	\$6.81							
13	N-NNNoRev	11							
14	P/A,i,N-NNNoRev	\$6.50							
15	P/F,i,NNNoRev	\$0.91							
16	Revenue Discount	\$5.90							
17	Output Variable								
18	Present Worth	\$105,293							

Eschenbach expressed the ranges as lower and upper limit percentages of the base case. To use SensIt, those percentages have been multiplied by the base case so that the OneExtreme and OtherExtreme values are in the same measurement units as the BaseCase values. For more information about choosing input values, refer to the “Tips for Many Inputs, One Output” section.

In Excel 2003, choose Tools | Sensitivity Analysis | Many Inputs, One Output. In Excel 2007 or 2010, choose Add-Ins | Sensitivity Analysis | Many Inputs, One Output.

Figure 4.6 SensIt Many Inputs, One Output Dialog Box

Ranges for Input Variables

Type a range reference, or point to the range (click and drag) containing text labels and the range containing numeric values that are inputs to your model. If the range is not contiguous, select the

first portion and then hold down the Control key while making the remaining selections. Alternatively, type a comma between each portion.

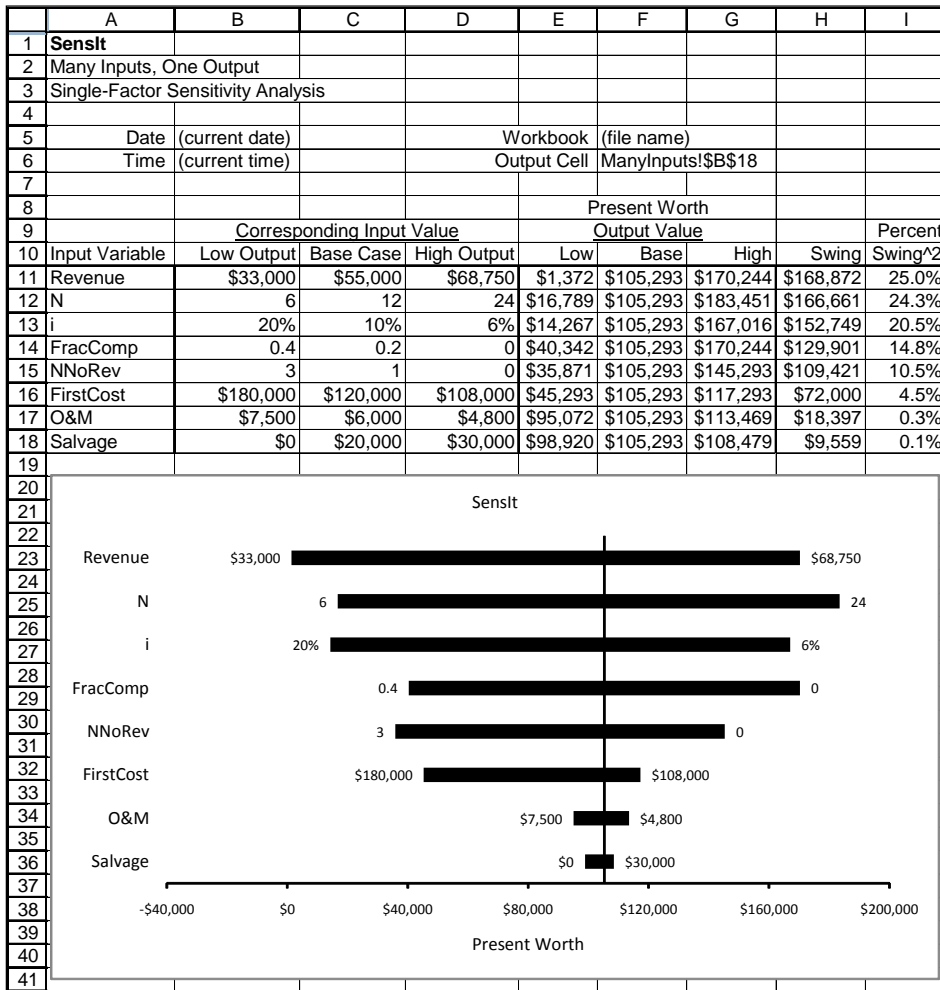
Cells for Output Variable

Type a cell reference, or point to the cell containing a text label and the cell containing a formula that's the output of your model.

Ranges for Input Values

Type a range reference, or point to the range (click and drag) containing numeric values for each of your model's inputs. You can make non-contiguous selections similar to the ranges for input variables. Be sure that all five range selections have the appropriate cells in the same order.

Figure 4.7 SensIt Tornado Numerical and Chart Output



For each input variable, SensIt sets all other input values at their Base case values, copies the One Extreme input value to the input variable cell, recalculates the worksheet, and copies the value of

the output variable cell to the table. The same steps are repeated using each Other Extreme input value. For each input variable, SensIt computes the range of the output variable values (the swing), sorts the table from largest swing down to smallest swing, and prepares a bar chart.

For each Low output value in column E, the corresponding input value is shown in column B. For each High output value in column G, the corresponding input value is shown in column D. The tornado chart shows single-factor sensitivity analysis, i.e., for each output value, only one input value is changed from its base case value. The tornado chart summarizes eight separate single-factor sensitivity analyses.

Insight The uncertainty about annual Revenue (the topmost bar in the tornado chart) is associated with the widest swing in Present Worth. O&M and Salvage have little effect on Present Worth.

4.5 CUSTOMIZING SENSITIVITY CHARTS

If the labels on the horizontal axis are numbers with many digits, some of the labels may wrap around so that some of the digits display below the others. There are many ways to remedy this anomaly.

(1) One way is to widen the chart. Click just inside the outer border of the chart so that eight chart handles are shown and then drag the middle chart handle on the left or right to widen the chart.

(2) Another way is to select the horizontal axis (click between the labels on the horizontal axis so that "Value (X) axis" appears in the name box in the upper left of Excel) and change to a smaller font size using the Font Size drop-down edit box on the Formatting tool bar. In Excel 2007 or 2010, select the axis, and change the font size in the Font area of the Home ribbon.

(3) Yet another way in Excel 2003 is to select the horizontal axis, choose Format | Selected Axis | Scale, and change the Minimum, Maximum, and Major unit. Select a "nice" number for the Minimum, specify a larger value for the Major unit, and use an appropriate value for the Maximum. In Excel 2007 or 2010, select the axis, select the Chart Tools Layout or Format ribbon, click Format Selection in the Current Selection area of the ribbon, click Axis Options in the Format Axis dialog box, and change the Minimum, Maximum, and Major unit.

(4) Finally, you can obtain more spacing on the horizontal axis by using a custom format. If you want to show values in thousands instead of the original units, in Excel 2003 select the horizontal axis, choose Format | Selected Axis | Number | Custom, and enter a comma at the end of the current format shown in the "Type:" edit box. In Excel 2007 or 2010, select the axis, select the Chart Tools Layout or Format ribbon, click Format Selection in the Current Selection area of the ribbon, click Number in the Format Axis dialog box, enter a comma at the end of the current format shown in the Format Code edit box, and click the Add button. For example, you might change the format from "\$#,##0" to "\$#,##0," and then modify the horizontal axis title to indicate that the axis values are in thousands.

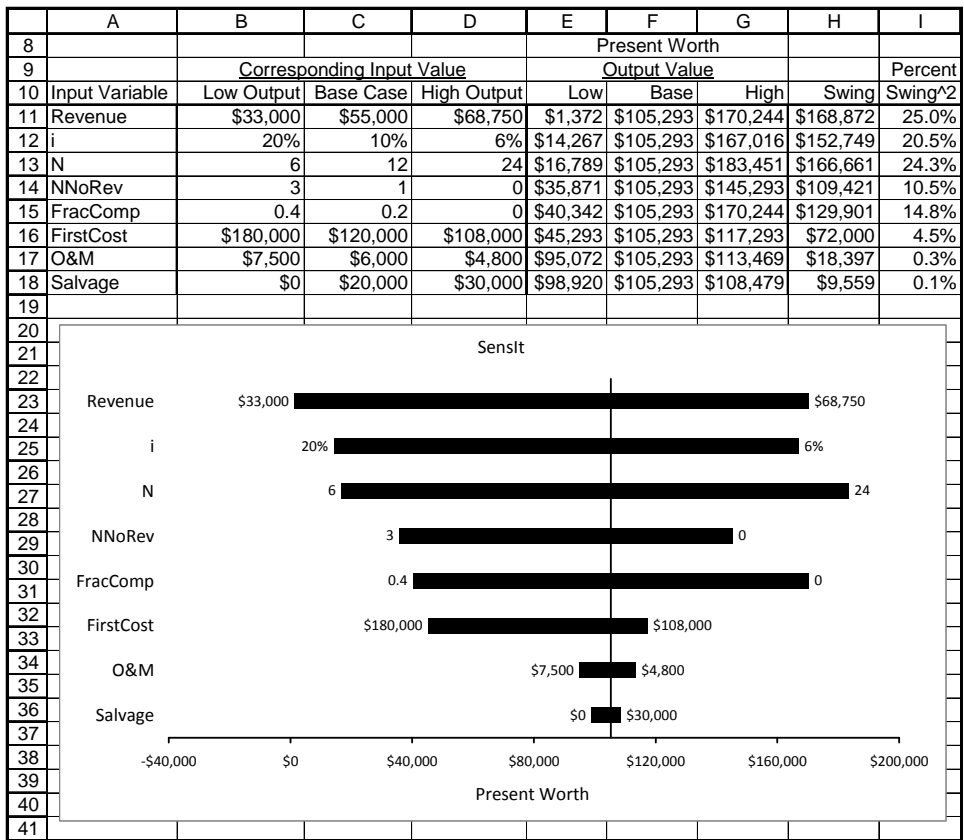
4.6 TORNADO SORTED BY DOWNSIDE RISK

The tornado chart is originally sorted by Swing. To sort by downside risk in Excel 2003, i.e., by the low output values, select the labels and data in cells A10:I18, choose Data | Sort, check that "Header row" is selected, select Sort by "Low" (not by "Low Output") Ascending, and click OK. If necessary, press F9 to refresh the chart.

In Excel 2007 or 2010, select the single row of labels and all data rows, on the Home ribbon in the Editing area choose Sort & Filter | Custom Sort, check that "My data has headers" is selected, select Column Sort by "Low," select Sort On Values, select Order Smallest to Largest, and click OK. If necessary, press F9 to refresh the chart.

The results using Excel 2003 SP3 are shown below.

Figure 4.8 SensIt Tornado Sorted by Downside Risk



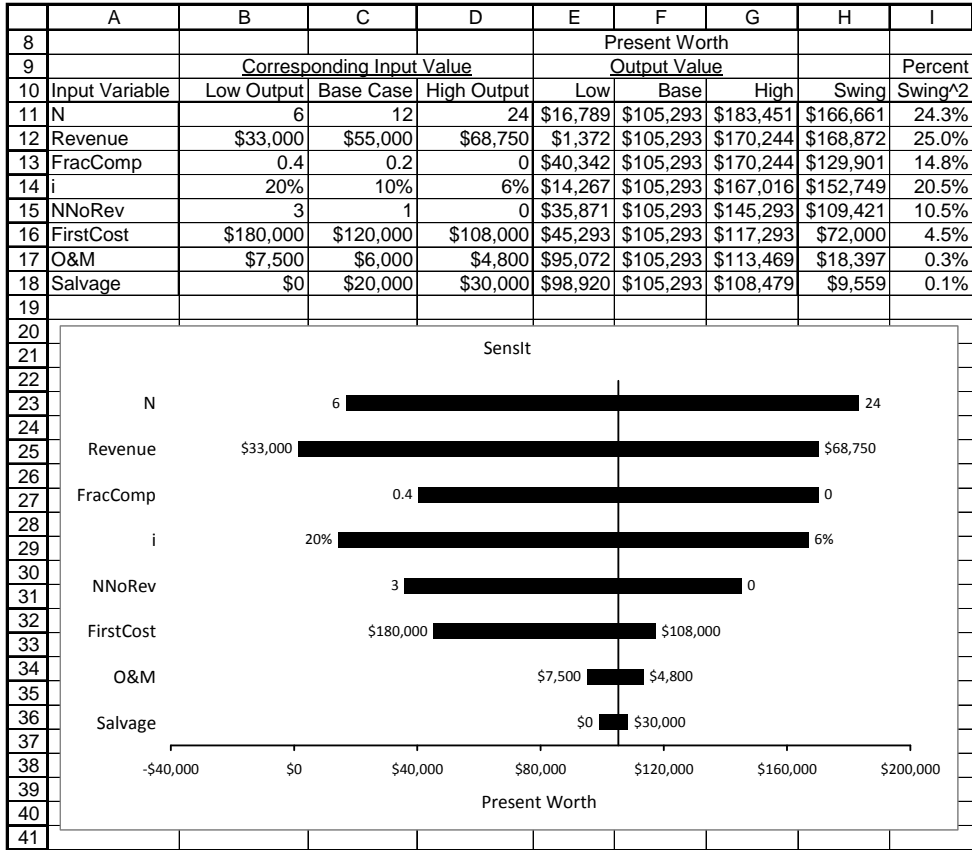
4.7 TORNADO SORTED BY UPSIDE POTENTIAL

To sort by upside potential in Excel 2003, i.e., by the high output values, select the data in cells A10:I18, choose Data | Sort, check that "Header row" is selected, select Sort by "High" (not by "High Output") Descending, and click OK. If necessary, press F9 to refresh the chart.

In Excel 2007 or 2010, select the single row of labels and all data rows, on the Home ribbon in the Editing area choose Sort & Filter | Custom Sort, check that "My data has headers" is selected, select Column Sort by "High," select Sort On Values, select Order Largest to Smallest, and click OK. If necessary, press F9 to refresh the chart.

The results using Excel 2003 SP3 are shown below.

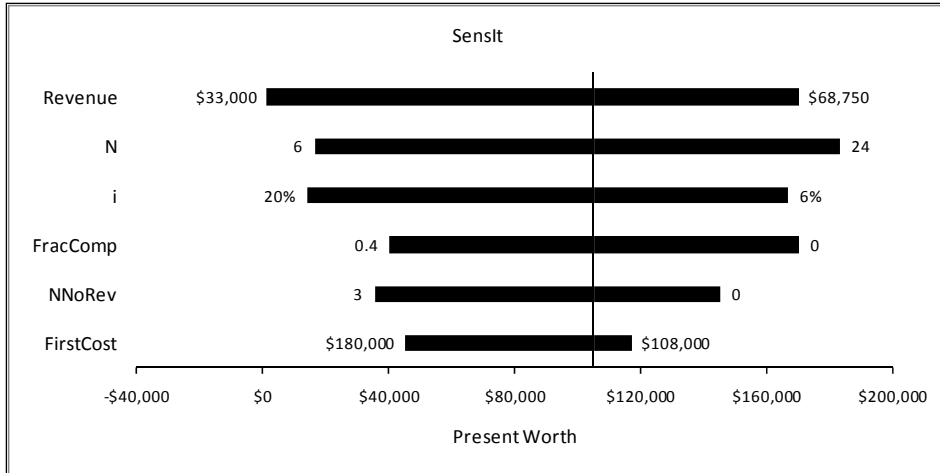
Figure 4.9 SensIt Tornado Sorted by Upside Potential



4.8 TORNADO SHOWING MAJOR UNCERTAINTIES

In some situations you may have twenty or more input variables and you wish to show the variation of only the top five or ten. To illustrate this modification, note that in this example the variation in Present Worth due to O&M and Salvage is minimal, so consider showing only the top six input variables, here sorted by swing. Click one of the bars on the left side of the vertical base case line to select Series 1 (shown at the right end of the SERIES function in the formula bar), and then click and drag the fill handle from A18 up to A16 and the fill handle from F18 up to F16. Click one of the bars on the right side of the vertical base case line to select Series 2, and then click and drag the fill handle from H18 up to H16. To resize the chart, click just inside its outer border and drag the bottom center fill handle upward. The resulting chart is shown below.

Figure 4.10 SensIt Tornado Showing Only Major Uncertainties

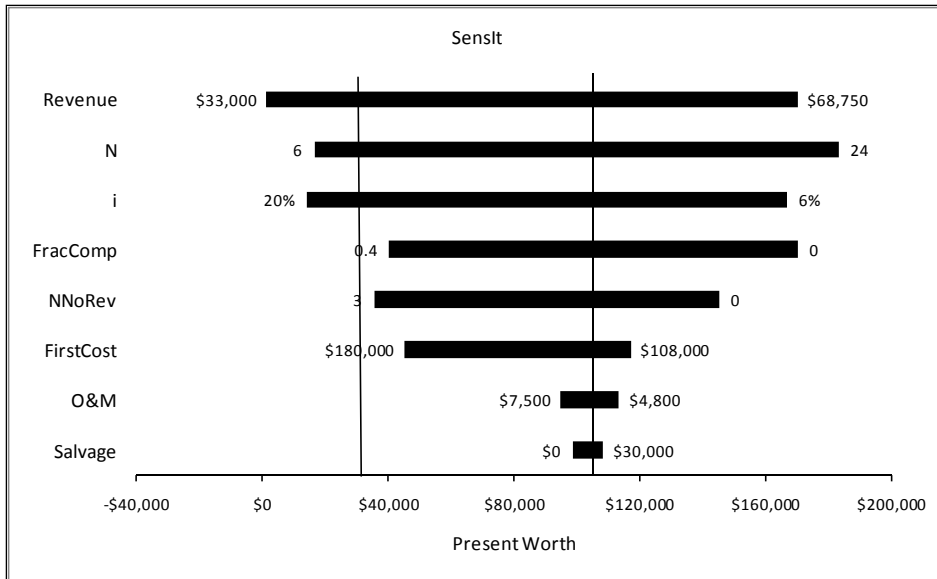


4.9 TORNADO WITH VERTICAL LINE

A tornado chart shows ranges for a single alternative. To compare with a second alternative, it is useful to show a vertical line representing the value of the performance measure for the second alternative. For example, the second alternative might be "Pursue a fixed cost contract," with present worth of \$30,000.

In Excel 2003, choose View | Toolbars, and click Drawing. Select the chart by clicking just inside its outer border so that the eight sizing handles appear on the border. Click the Line icon on the Drawing toolbar.

In Excel 2007 or 2010, select the chart, on the Insert ribbon in the Illustrations area click Shapes, and in the Lines area click the Line icon. On the tornado chart, click at the \$30,000 value on the horizontal axis and drag upward to above the top horizontal bar. Before releasing the mouse button, position the top end of the line so that it is vertical. The result is shown below.

Figure 4.11 Tornado Chart with Vertical Line

To format the vertical line in Excel 2003, select it, and choose Format. For the line shown above, the Weight was changed from 0.75 to 1.5 on the Format AutoShape Colors and Lines tab.

In Excel 2007 or 2010, select the vertical line, on the Drawing Tools Format ribbon choose Shape Outline | Weight 1 pt.

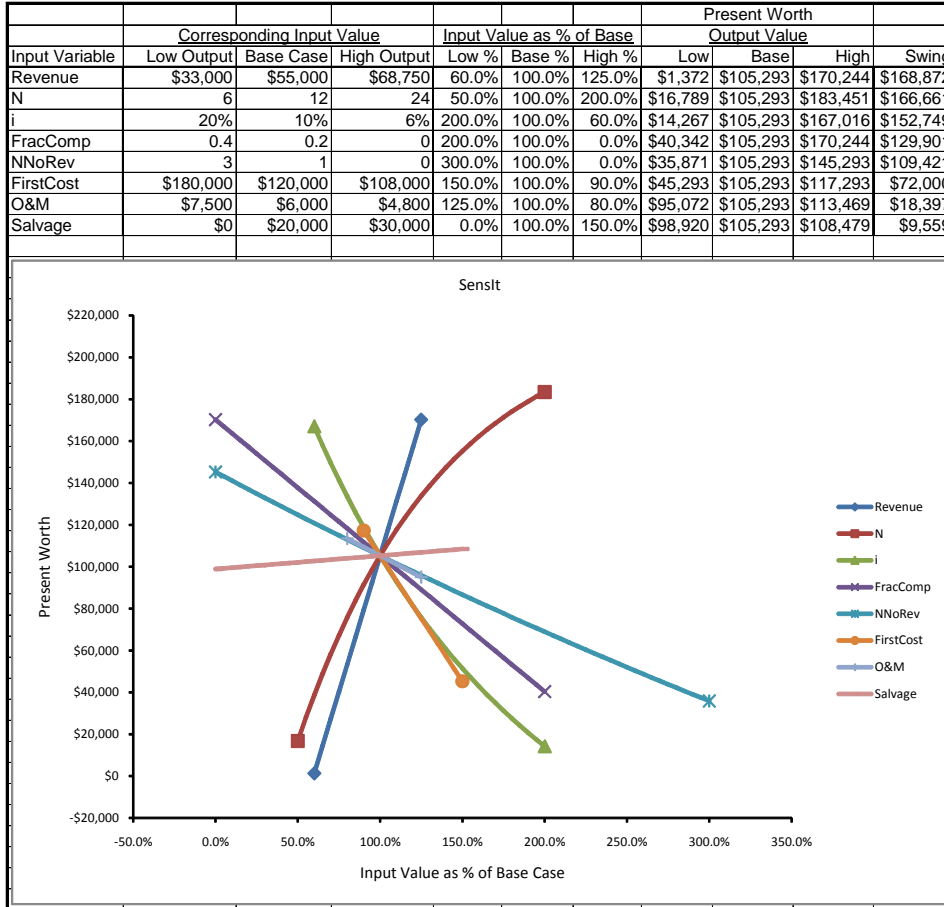
Insight In this example, we note that Revenue, N, and i are critical input assumptions. These are the three input variables that could result in Present Worth less than the "Pursue a fixed cost contract" alternative. However, three other input assumptions are nearly as critical. We should focus on these six inputs, either by trying to control the value, by gathering more information to reduce the uncertainty (the width of the bars), or by treating the uncertainty explicitly using probability assessments and subsequent Monte Carlo simulation. We should not be as concerned about the O&M and Salvage values.

4.10 MANY INPUTS, ONE OUTPUT SPIDER

In addition to the tornado chart, SensIt's Many Inputs, One Output option also creates a spider chart to show how your model's output depends on the percentage changes for each of the model's input variables.

SensIt Spider uses the same base case and extreme input values as the single-factor sensitivity analysis for the tornado chart, but you can specify a Step Percent for evaluating the model at intermediate values. The results are shown with each input value expressed as a percentage of the base case input value. If the base case value for an input variable is zero, that variable is not included in the Spider analysis (because it would not be possible to express an extreme or intermediate input value as a percentage of the zero base case value). SensIt prepares an XY (Scatter) chart where the horizontal axis is each input value as a percentage of the base case input value and the vertical axis is the associated model output value.

Figure 4.12 SensIt Spider Numerical and Chart Output



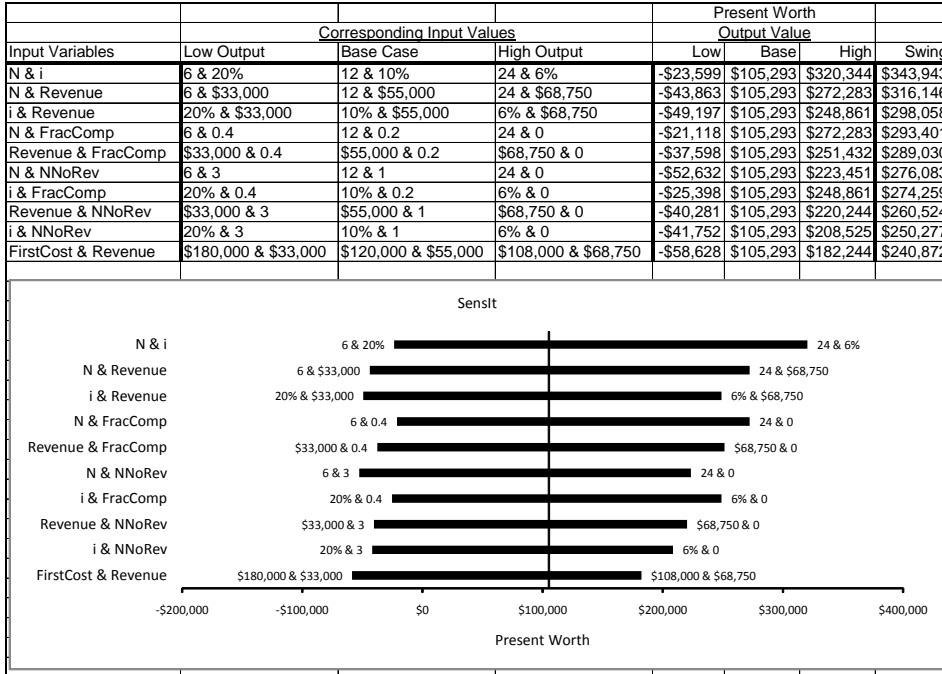
Insight On a spider chart, lines that are nearly horizontal generally indicate an input variable where small percentage changes do not have much affect on the output value. Lines that are more vertical indicate an input variable where small percentage changes have a greater affect on the output value. Unlike the tornado chart, the spider chart makes it easy to identify nonlinear relationships.

4.11 TWO-FACTOR TORNADO CHART

SensIt can also produce a two-factor tornado chart, where SensIt considers each pair of input variables. If N is the number of input variables, there are $N*(N-1)/2$ pairs to evaluate. For each pair, SensIt considers all nine combinations of the One Extreme, Base Case, and Other Extreme input values for the two variables. For the calculation of output for each combination, the other input variables are kept at their Base Case values. For each pair, SensIt summarizes the calculations by showing the combinations of input values that produce the lowest and highest output value. Finally, the pairs are sorted by swing, and a tornado chart is created.

The example has $N=8$ input variables, so there are $N*(N-1)/2 = 8*(8-1)/2 = 28$ pairs to evaluate. SensIt's output shows data rows and chart bars for all 28 pairs. To save space, the figure shows numerical results for only the top ten (by hiding rows for the other 18 pairs and resizing the chart).

Figure 4.13 SensIt Two-Factor Tornado Partial Output



Insight The single-factor analysis produces Present Worth values between \$1,372 and \$170,244. The two-factor analysis produces Present Worth values between -\$58,628 and \$320,344. If we used Monte Carlo simulation, we would observe even more variation in the output, equivalent to an eight-factor sensitivity analysis.

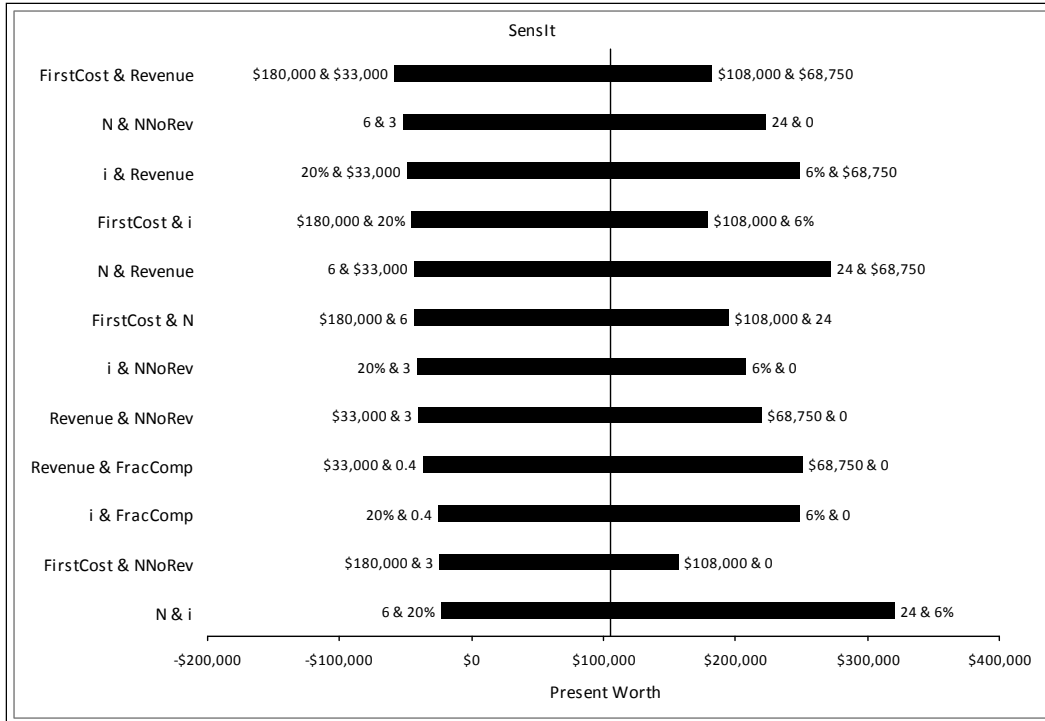
The single-factor downside-risk tornado chart shows that the lowest Present Worth is for the Revenue input assumption, followed by i, N, NNoRev, FracComp, and FirstCost. The two-factor analysis shows that the FirstCost and Revenue pair could produce the lowest Present Worth. So it is particularly important to find a way to avoid the high FirstCost and low Revenue combination, even though the single-factor analysis indicates that FirstCost by itself is not so important.

4.12 TWO-FACTOR DOWNSIDE RISK

In the following chart the two-factor results are sorted by the "Low" output value to show downside risk. Instead of showing all 28 results, the figure shows only the top twelve (by hiding rows for the other 16 pairs of inputs and resizing the chart).

The results using Excel 2003 SP3 are shown below.

Figure 4.14 SensIt Two-Factor Tornado Sorted by Downside Risk

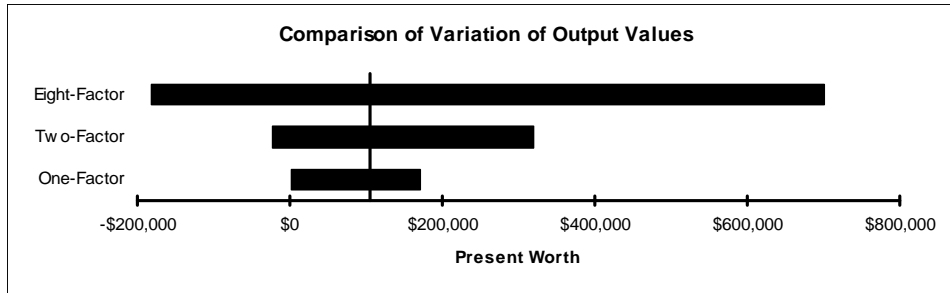


Insight From the one-factor analyses, we note that the inputs associated with maximum swing are Revenue, N, and i, in that order. When sorted by downside risk, the ordered inputs are Revenue, i, and N. Although those three inputs are included in the top ten pairs of the two-factor analysis shown above, the top two pairs indicate that FirstCost is an important input when combined with Revenue, and NNoRev is an important input when combined with N.

4.13 COMPARISON WITH OVERALL VARIATION

Using swing as the criterion for measuring variation of the output value, the single most important input variable is Revenue with output variation from \$1,372 to \$170,244, a swing of \$168,872. With two-factor sensitivity analysis, the two most important input variables are N and i with output variation from -\$23,599 to \$320,344, a swing of \$343,943.

When all input assumptions are varied (computations are not shown here), the output varies from -\$180,805 to \$702,005, a swing of \$882,810. The figure shows the output variation for these three situations. For example, if you use triangular density functions for each input variable and perform Monte Carlo simulation with a very large number of trials, the results will show similar variation.

Figure 4.15 Comparison of Variation of Output Values

4.14 TIPS FOR MANY INPUTS, ONE OUTPUT

When defining the high and low cases for each variable, it is important to be consistent so that the "high" cases are all equally high and the "low" cases are all equally low. This will ensure that the output results can be meaningfully compared.

For example, if you are thinking about uncertainty using probability and very extreme values are possible but with low probability of occurrence, you might take all of the base case values to be estimates of the mean of the input variable, take low cases to be values such there is a 1-in-10 chance of the variable being below this amount, and take the high cases to be values such that there is a 1-in-10 chance of the variable being above this amount. Or, you might use the 5th and 95th percentiles for each of the input variables.

Alternatively, in some situations the values for each input variable may have lower and upper bounds, so you may specify low and high values that are the absolute lowest and highest possible values.

When you click OK, SensIt sets all of the input variables to their base-case values and records the output value. Then SensIt goes through each of the input variables one at a time, plugs the low-case value into the input cell, and records the value in the output cell. It then repeats the process for the high case. For each substitution, all input values are kept at their base-case values except for the single input value that is set at its low or high value. SensIt then produces a spreadsheet that lists the numerical results as shown in columns E, F, and G of the worksheet with the tornado chart.

In the worksheet, the variables are sorted by their "swing" -- the absolute value of the difference between the output values in the low and high cases. "Swing" serves as a rough measure of the impact of each input variable. The rows of numerical output are sorted from highest swing at the top down to lowest swing at the bottom. Then SensIt creates a bar chart of the sorted data.

In general, you should focus your modeling efforts on those variables with the greatest impact on the value measure.

If your model has input variables that are discrete or categorical, you should create multiple tornado charts using different base case values of that input variable. For example, if your model has an input variable "Government Regulation" that has possible values 0 (zero) or 1, the low and high values will be 0 and 1, but you should run one tornado chart with base case = 0 and another tornado chart with base case = 1.

4.15 INTERPRETATION OF PERCENT SWING²

A tornado chart displays the results of single-factor sensitivity analysis for an alternative. For a risk averse decision maker, the certain equivalent of an alternative is less than the expected value (i.e., probability-weighted average) of the payoff distribution. Each Percent Swing² value indicates how the difference between the expected value and the certain equivalent is affected by the uncertainty of a specific input variable. These approximations are based on the following assumptions:

1. Exponential Utility. The decision maker's attitude toward risk is described by an exponential utility function, with a Risk Tolerance parameter indicating the degree of risk aversion. In this case the certain equivalent of a payoff distribution can be approximated by

$$\text{Certain Equivalent} = \text{Expected Value} - \text{Variance}/(2 * \text{Risk Tolerance})$$

The variance of each input variable contributes to the variance of the payoff distribution, which determines how much the certain equivalent is less than the expected value.

2. Independence. The input variables should be probabilistically independent, in which case the variance of the payoff distribution is the sum of the variances of the input variables (for small variations and perhaps a weighted sum, depending on the payoff model). This assumption of independence is also necessary for single-factor sensitivity analysis to provide meaningful information.
3. Symmetric Probability Distributions. The approximation of certain equivalent based on the variance is most accurate for uncertain input assumptions specified by symmetric, bell-shaped probability density functions. As an example, for an input distribution that has a normal probability distribution, the standard deviation is proportional to the width (high extreme minus low extreme) specified as inputs to the single-factor sensitivity analysis, and the variance is proportional to the width squared. For an unbounded symmetric probability distribution, the extremes are often specified using the 0.05 and 0.95 fractiles or the 0.10 and 0.90 fractiles.

For a more extensive discussion, see McNamee and Celona (2001).

4.16 ACKNOWLEDGEMENT

The original version of SensIt was written using the Excel 4 XLM macro programming language by Mike Middleton of the University of San Francisco and Jim Smith of Duke University. The current version with spider and two-factor tornado charts was completely rewritten by Mike Middleton during 2005–2010 using the Excel 2000 VBA programming language.

4.17 REFERENCES

Eschenbach, Ted G. "Spiderplots versus Tornado Diagrams for Sensitivity Analysis," *Interfaces* (An International Journal of The Institute of Management Sciences and the Operations Research Society of America), Special Issue: Decision and Risk Analysis, Vol. 22, No. 6 (Nov.-Dec. 1992), pp. 40-46.

McNamee, Peter, John Celona. *Decision Analysis for the Professional*. 3rd ed. Menlo Park, CA: SmartOrg, 2001.