

# Monte Carlo Simulation Using RiskSim

# 10

## 10.1 RISKSIM OVERVIEW

RiskSim is a Monte Carlo Simulation add-in for Microsoft Excel 2000–2010 (Windows) and Microsoft Excel 2004 & 2011 (Macintosh).

RiskSim provides random number generator functions as inputs for your model, automates Monte Carlo simulation, and creates charts. Your spreadsheet model may include various uncontrollable uncertainties as input assumptions (e.g., demand for a new product, uncertain variable cost of production, competitor reaction), and you can use simulation to determine the uncertainty associated with the model's output (e.g., annual profit). RiskSim automates the simulation by trying hundreds of what-ifs consistent with your assessment of the uncertainties.

To use RiskSim, you

- (1) create a spreadsheet model
- (2) optionally use SensIt to identify critical inputs
- (3) enter one of RiskSim's fourteen random number generator functions in each input cell of your model
- (4) in Excel 2007 or 2010, choose Add-Ins | Risk Simulation | One Output; in Excel 2003 and earlier versions, choose Tools | Risk Simulation | One Output from Excel's menu
- (5) specify the model output cell and the number of what-if trials
- (6) interpret RiskSim's histogram and cumulative distribution charts.

RiskSim facilitates Monte Carlo simulation by providing:

- Fourteen random number generator functions
- Ability to set the seed for random number generation
- Automatic repeated sampling for simulation
- Frequency distribution of simulation results
- Histogram and cumulative distribution charts

All of RiskSim's functionality, including its built-in help, is a part of the RiskSim XLA file. There is no separate setup file or help file. When you use RiskSim on a Windows computer, it does not create any Windows Registry entries (although Excel may use such entries to keep track of its add-ins).

## 10.2 USING RISKSIM FUNCTIONS

RiskSim adds fourteen random number generator functions to Excel. You can use these functions as inputs to your model by typing in a worksheet cell or by using the Function Wizard. From the Insert menu choose Function, or click the Function Wizard button. RiskSim's functions are listed in a User Defined category. The fourteen functions are:

```

RANDBINOMIAL(trials,probability_s)
RANDBIVARNORMAL(mean1,stdev1,mean2,stdev2,correl12)
RANDCUMULATIVE(value_cumulative_table)
RANDDISCRETE(value_discrete_table)
RANDEXPONENTIAL(lambda)
RANDINTEGER(bottom,top)
RANDLOGNORMAL(Mean,StDev)
RANDNORMAL(mean,standard_dev)
RANDPOISSON(mean)
RANDSAMPLE(population)
RANDTRIANGULAR(minimum,most_likely,maximum)
RANDTRUNCBIVARNORMAL(mean1,stdev1,mean2,stdev2,correl12,
min1,max1,min2,max2)
RANDTRUNCNORMAL(Mean,StDev,MinValue,MaxValue))
RANDUNIFORM(minimum,maximum)

```

RiskSim's RAND... functions include extensive error checking of arguments. After verifying that the functions are working properly, you may want to substitute RiskSim's FAST... functions which have minimal error checking and therefore run faster. From the Edit menu choose Replace; in the Replace dialog box, type =RAND in the "Find What" edit box, type =FAST in the "Replace with" edit box, and click the Replace All button.

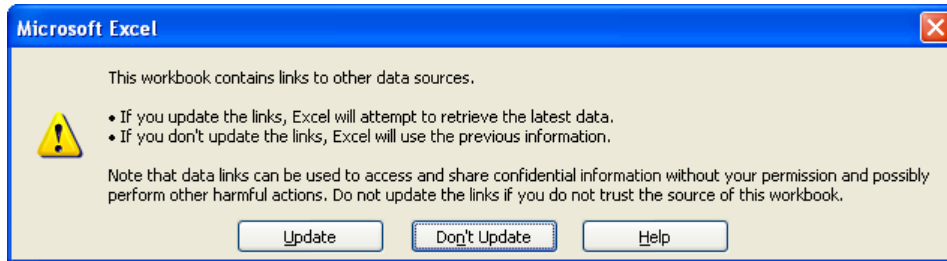
## 10.3 UPDATING LINKS TO RISKSIM FUNCTIONS

When you insert a RiskSim random number generator function in a worksheet cell, the function is linked to the disk location of the RiskSim XLA file you are currently using. During the current Excel session, the formula bar shows only the name of the RiskSim function. But when you save and close the workbook, Excel saves the complete path to the disk location of RiskSim function. For example, after closing and reopening the workbook, the formula bar might show C:\MyAddIns\risk240s.xla\RandNormal(100, 10). This is standard behavior for Excel user defined functions like the ones contained in the RiskSim XLA file.

When you open the workbook, Excel looks for the RiskSim XLA file using the saved path. If Excel cannot find the RiskSim XLA file at the saved path location (e.g., if you deleted the RiskSim XLA file from the C:\MyAddIns folder or if you opened the workbook on another

computer where the RiskSim XLA file is not located at the same path), Excel displays a dialog box like the one shown below.

**Figure 10.1** Excel 2003 Warning To Update Links

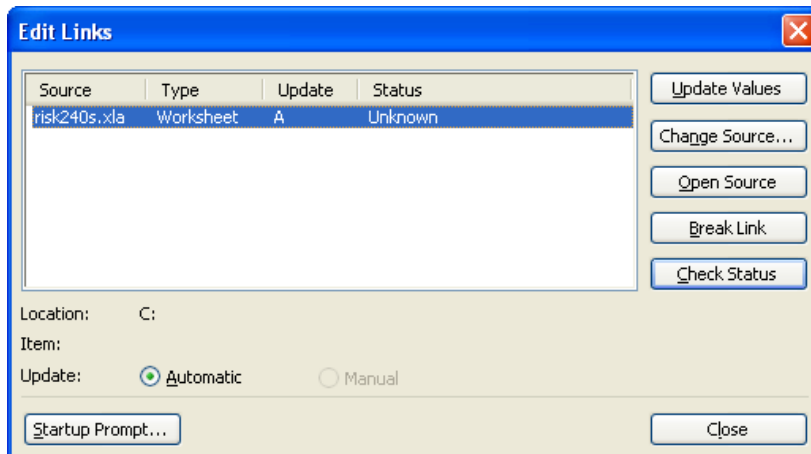


If you see this dialog box or a similar warning when you open an Excel file, choose the "Don't Update" option. The workbook will be opened, but any cell containing a reference to a RiskSim function will display the #NAME? or similar error code.

To update the links after the workbook is open, be sure that a RiskSim XLA file is open. Then choose Edit | Links to see the dialog box shown below.

In Excel 2007 or 2010, if you open a workbook with RiskSim functions referring to a RiskSim file location that no longer exists, you may see a warning. In Excel 2007, if you click the Options button of the security warning, you can click OK in the Security Options dialog box to dismiss the warning. Before you update the links, be sure that the RiskSim XLA file is open. In Excel 2007, choose Office Button | Prepare | Edit Links to Files to see the dialog box shown below. In Excel 2010, choose File | Info | Edit Links to Files to see the Edit Links dialog box.

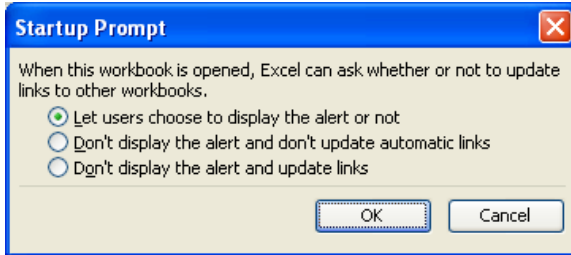
**Figure 10.2** Edit Links Dialog Box



To update the links, click the Change Source button. A file browser window will open, where you can navigate to the RiskSim XLA file that is open. After you select the file using the file browser, click OK. Back in the Edit Links dialog box, click the Close button.

In Excel 2003 the Edit Links dialog box has a Startup Prompt button. To avoid possible problems when Excel tries to automatically update links while a file is being opened, we recommend the default "Let users choose to display the alert or not."

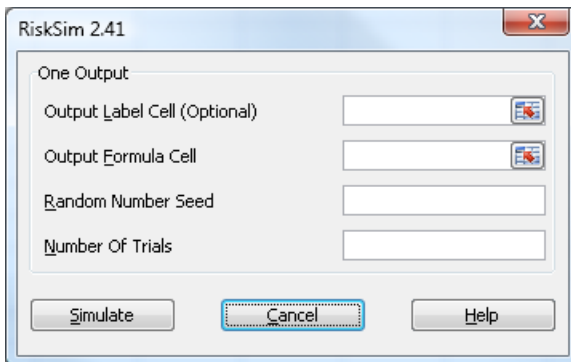
**Figure 10.3** Excel 2003 Startup Prompt Dialog Box



## 10.4 MONTE CARLO SIMULATION

After specifying random number generator functions as inputs to your model, from the Tools choose Risk Simulation | One Output.

**Figure 10.4** RiskSim Dialog Box



Optionally, select the "Output Label Cell" edit box, and point or type a reference to a cell containing the name of the model output (for example, a cell whose contents is the text label "Net Profit").

Select the "Output Formula Cell" edit box, and point to a single cell on your worksheet or type a cell reference. The output cell of your model must contain a formula that depends, usually indirectly, on the model inputs determined by the random number generator functions.

Leave the Random Number Seed unchanged, or select the "Random Number Seed" edit box, and type a number between 1 and 2,147,483,647. Use an integer value without commas or other separators.

Select the "Number Of Trials" edit box, and type an integer value between 2 and 32,000. This value, sometimes called the sample size or number of iterations, specifies the number of times the worksheet will be recalculated to determine output values of your model.

## 10.5 RANDOM NUMBER SEED

The "Random Number Seed" edit box on the RiskSim dialog box allows you to set the seed for RiskSim's random number generator functions. The seed must be an integer in the range 1 through 2,147,483,647. RiskSim's random number generator functions depend on RiskSim's own uniform random number function that is completely independent of Excel's built-in RAND().

Random numbers generated by the computer are actually pseudo-random. The numbers appear to be random, and they pass various statistical tests for randomness. But they are actually calculated by an algorithm where each random number depends on the previous random number. Such an algorithm generates a repeatable sequence. The seed specifies where the algorithm starts in the sequence.

A Monte Carlo simulation model usually has uncontrollable inputs (uncertain quantities using random number generator functions), controllable inputs (decision variables that have fixed values for a particular set of simulation iterations), and an output variable (a performance measure or operating characteristic of the system).

For example, a simple queuing system model may have an uncertain arrival pattern, a controllable number of servers, and total cost (waiting time plus server cost) as output. To evaluate a different number of servers, you would specify the same seed before generating the uncertain arrivals. Then the variation in total cost should depend on the different number of servers, not on the particular sequence of random numbers that generates the arrivals.

## 10.6 ONE-OUTPUT EXAMPLE

In this example the decision maker has described his subjective uncertainty using normal, triangular, and discrete probability distributions.

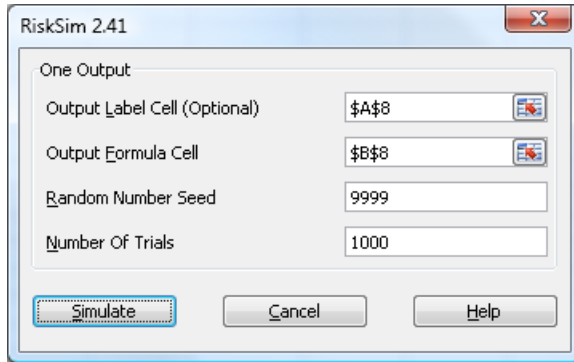
**Figure 10.5** One-Output Example Model Display

	A	B	C	D	E	F	G	H
1	Software Decision Analysis							
2								
3	Unit Price	\$29		Price is controllable and constant.				
4	Units Sold	739		Normal	Mean = 700, StDev = 100			
5	Unit Variable Cost	\$8.05		Triangular	Min = \$6, Mode = \$8, Max = \$11			
6	Fixed Costs	\$12,000		Discrete	Value	Probability		
7					\$10,000	0.25		
8	Net Cash Flow	\$3,485			\$12,000	0.50		
9					\$15,000	0.25		

**Figure 10.6** One-Output Example Model Formulas

	A	B
1	Software Decision Analysis	
2		
3	Unit Price	\$29
4	Units Sold	=INT(RANDNORMAL(700,100))
5	Unit Variable Cost	=RANDTRIANGULAR(6,8,11)
6	Fixed Costs	=RANDDISCRETE(E7:F9)
7		
8	Net Cash Flow	=B4*(B3-B5)-B6

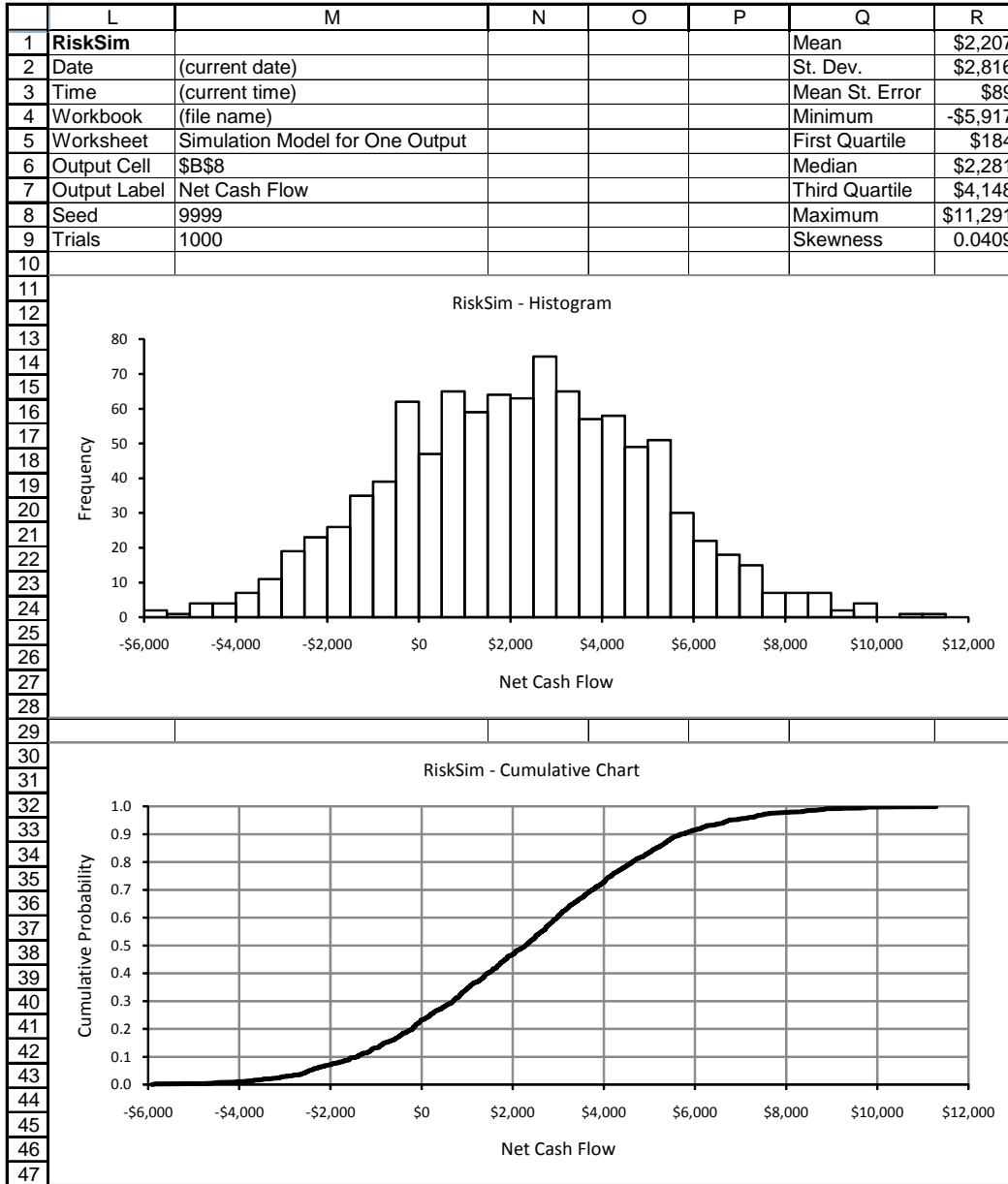
**Figure 10.7** RiskSim Dialog Box for One-Output Example



## 10.7 RISKSIM OUTPUT FOR ONE-OUTPUT EXAMPLE

When you click the Simulate button, RiskSim creates a new worksheet in your Excel workbook named "RiskSim Summary 1." A summary of your inputs and the output is shown in cells L1:R9 with the accompanying histogram and cumulative distribution charts.

**Figure 10.8** RiskSim Summary Output for One-Output Example



The histogram is based on the frequency distribution in columns I:J. The cumulative distribution is based on the sorted output values in column C and the cumulative probabilities in column D.

**Figure 10.9** RiskSim Numerical Output for One-Output Example

	A	B	C	D	E	F	G	H	I	J
1	Trial	Net Cash Flow	Sorted	Cumulative		Percent	Percentile		Upper Limit	Frequency
2	1	\$2,815	-\$5,917	0.0005		0%	-\$5,917		-\$6,000	0
3	2	-\$1,381	-\$5,860	0.0015		5%	-\$2,459		-\$5,500	2
4	3	-\$496	-\$5,422	0.0025		10%	-\$1,405		-\$5,000	1
5	4	-\$866	-\$4,702	0.0035		15%	-\$809		-\$4,500	4
6	5	\$5,795	-\$4,646	0.0045		20%	-\$196		-\$4,000	4
7	6	-\$3,320	-\$4,601	0.0055		25%	\$184		-\$3,500	7
8	7	\$3,295	-\$4,513	0.0065		30%	\$694		-\$3,000	11
9	8	\$8,360	-\$4,439	0.0075		35%	\$1,032		-\$2,500	19
10	9	-\$2,157	-\$4,057	0.0085		40%	\$1,442		-\$2,000	23
11	10	\$4,313	-\$4,037	0.0095		45%	\$1,835		-\$1,500	26
12	11	-\$1,375	-\$4,027	0.0105		50%	\$2,281		-\$1,000	35
13	12	\$2,198	-\$3,884	0.0115		55%	\$2,634		-\$500	39
14	13	-\$744	-\$3,846	0.0125		60%	\$2,972		\$0	62
15	14	\$3,104	-\$3,715	0.0135		65%	\$3,324		\$500	47
16	15	-\$2,814	-\$3,684	0.0145		70%	\$3,737		\$1,000	65
17	16	\$4,165	-\$3,676	0.0155		75%	\$4,148		\$1,500	59
18	17	-\$2,575	-\$3,561	0.0165		80%	\$4,617		\$2,000	64
19	18	\$2,643	-\$3,525	0.0175		85%	\$5,142		\$2,500	63
20	19	-\$3,676	-\$3,492	0.0185		90%	\$5,673		\$3,000	75
21	20	\$2,186	-\$3,468	0.0195		95%	\$6,736		\$3,500	65
22	21	-\$3,492	-\$3,320	0.0205		100%	\$11,291		\$4,000	57
23	22	-\$847	-\$3,285	0.0215					\$4,500	58
24	23	\$3,335	-\$3,250	0.0225					\$5,000	49
25	24	\$4,641	-\$3,178	0.0235					\$5,500	51
26	25	\$4,814	-\$3,118	0.0245					\$6,000	30
27	26	\$536	-\$3,104	0.0255					\$6,500	22
28	27	-\$909	-\$3,071	0.0265					\$7,000	18
29	28	\$4,477	-\$3,065	0.0275					\$7,500	15
30	29	-\$597	-\$3,054	0.0285					\$8,000	7
31	30	\$4,018	-\$2,960	0.0295					\$8,500	7
32	31	\$3,012	-\$2,957	0.0305					\$9,000	7
33	32	\$4,590	-\$2,830	0.0315					\$9,500	2
34	33	\$174	-\$2,820	0.0325					\$10,000	4
35	34	\$3,257	-\$2,814	0.0335					\$10,500	0
36	35	\$6,952	-\$2,795	0.0345					\$11,000	1
37	36	\$6,252	-\$2,671	0.0355					\$11,500	1
38	37	\$4,017	-\$2,654	0.0365					\$12,000	0
39	38	\$5,386	-\$2,638	0.0375						

In column D, the cumulative probabilities start at  $1/(2*N)$ , where N is the number of trials, and increase by  $1/N$ . The rationale is that the lowest ranked output value of the sampled values is an estimate of the population's values in the range from 0 to  $1/N$ , and the lowest ranked value is associated with the median of that range.

Column B contains the original sampled output values.

Columns F:G show percentiles based on Excel's PERCENTILE worksheet function. Refer to Excel's online help for the interpolation method used by the PERCENTILE function.

The summary measures in columns Q:R are also based on Excel worksheet functions: AVERAGE, STDEV, QUARTILE, and SKEW.

## 10.8 CUSTOMIZING RISKSIM CHARTS

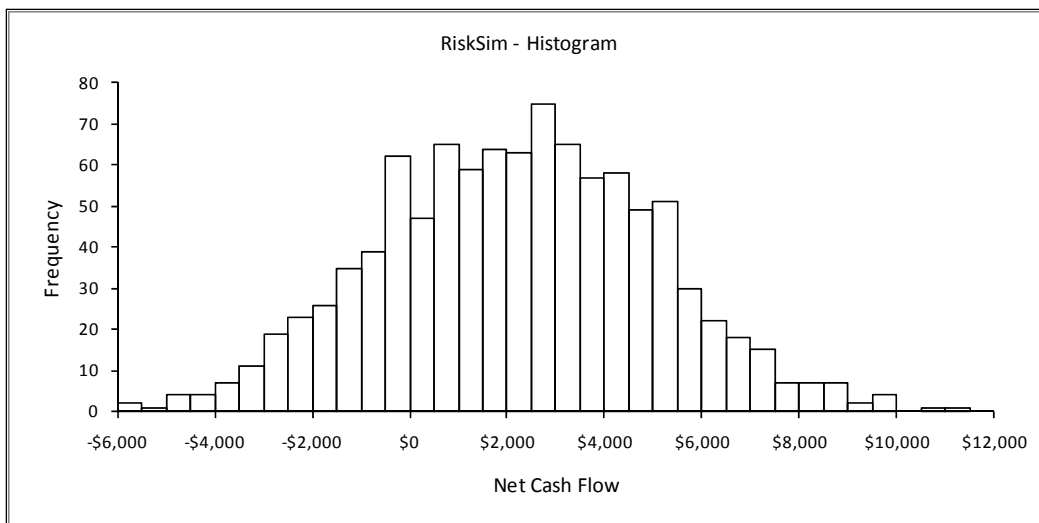
If the labels on the horizontal axis are numbers with many digits, some of the labels may wrap around so that some of the digits display below the others. One way to remedy this anomaly is to

widen the chart (click just inside the outer border of the chart so that eight chart handles are shown and then drag the middle chart handle on the left or right to widen the chart). Another way is to select the horizontal axis (click between the labels on the horizontal axis so that "Value (X) axis" appears in the name box in the upper left of Excel) and change to a smaller font size using the Font Size drop-down edit box on the the Formatting tool bar.

The histogram chart is a combination chart using a column chart type for the vertical bars and an XY (Scatter) chart type for the horizontal axis. The two chart types align properly as long as the horizontal axis retains the same minimum and maximum values.

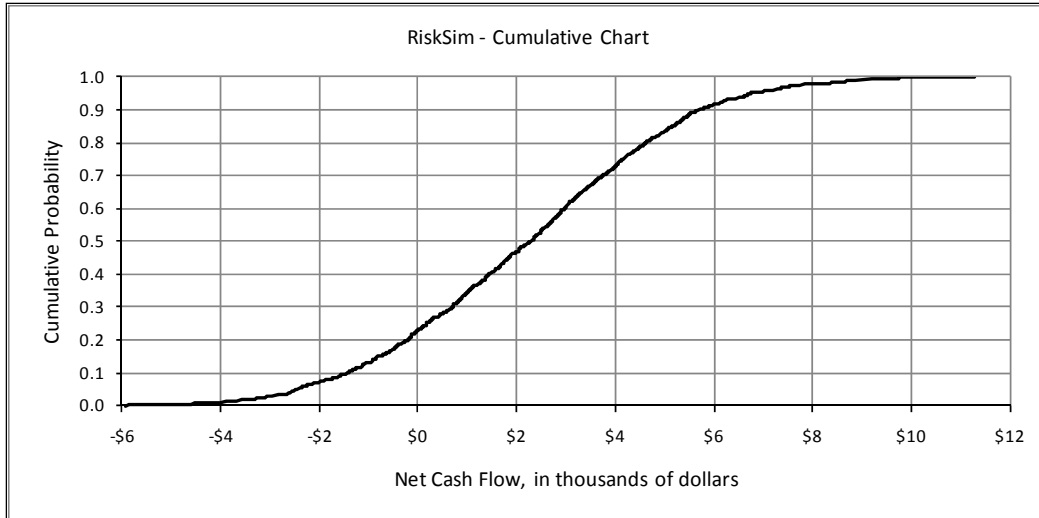
For example, if you want more spacing between the dollar labels on the horizontal axis, select the horizontal axis (so that "Value (X) axis" appears in the name box in the upper left of Excel), choose Format | Selected Axis | Scale, and change the "Major unit" from 1000 to 2000. In Excel 2007 or 2010, select the axis, right-click, and choose Format Axis | Axis Options. Do not change the Minimum = -6000 or the Maximum = 12000. The histogram appears as shown below.

**Figure 10.10** Original Histogram With Modified Horizontal Axis Major Unit



The cumulative chart is a standard XY (Scatter) chart type, so you can change the major unit as described above, but you can also change the minimum and maximum without affecting the integrity of the chart.

Another way to obtain more spacing on the horizontal axis of the histogram or cumulative chart is to use a custom format. For example, if you want to show values in thousands instead of the original units, select the horizontal axis (click between the labels on the horizontal axis so that "Value (X) axis" appears in the name box in the upper left of Excel), choose Format | Selected Axis | Number | Custom, and enter a comma at the end of the current format shown in the "Type:" edit box. In Excel 2007 or 2010, right-click the horizontal axis, choose Format Axis | Number, change the Format Code, and click the Add button. After changing the original format "\$#,##0" to "\$#,##0," and modifying the horizontal axis title, the cumulative chart appears as shown below.

**Figure 10.11** Original Cumulative Chart With Horizontal Axis Custom Format

## 10.9 RANDOM NUMBER GENERATOR FUNCTIONS

### RandBinomial

Returns a random value from a binomial distribution. The binomial distribution can model a process with a fixed number of trials where the outcome of each trial is a success or failure, the trials are independent, and the probability of success is constant. `RANDBINOMIAL` counts the total number of successes for the specified number of trials. If  $n$  is the number of trials, the possible values for `RANDBINOMIAL` are the non-negative integers  $0, 1, \dots, n$ .

`RANDBINOMIAL` Syntax: `RANDBINOMIAL(trials,probability_s)`

Trials (often denoted  $n$ ) is the number of independent trials.

Probability\_s (often denoted  $p$ ) is the probability of success on each trial.

#### `RANDBINOMIAL` Remarks

Returns #N/A if there are too few or too many arguments.

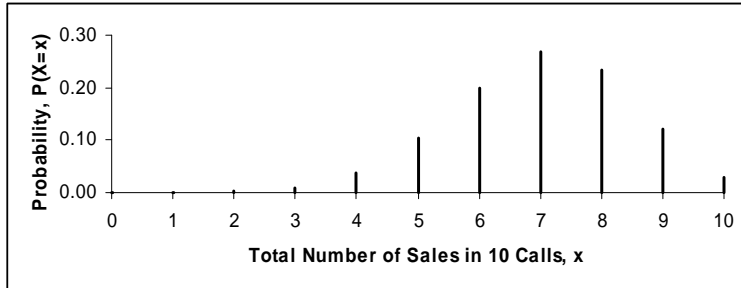
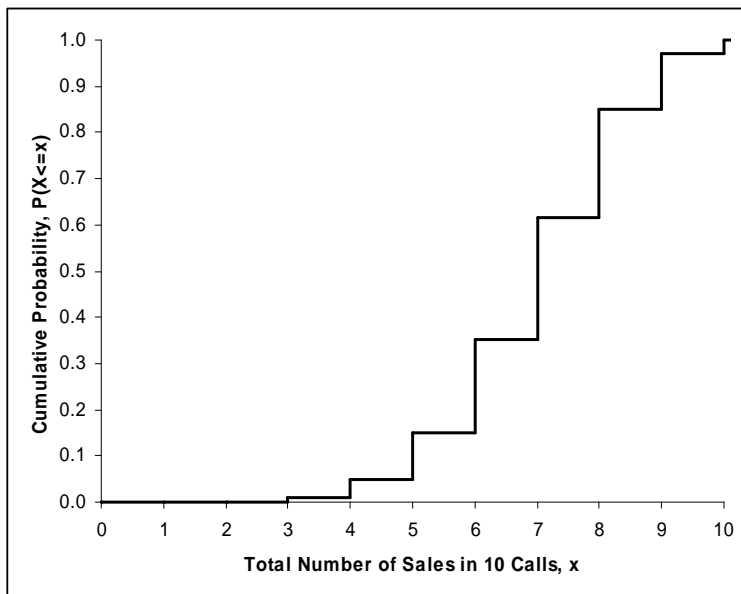
Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if trials is non-integer or less than one, or probability\_s is less than zero or more than one.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

#### `RANDBINOMIAL` Example

A salesperson makes ten unsolicited calls per day, where the probability of making a sale on each call is 70 percent. The uncertain total number of sales in one day is `=RANDBINOMIAL(10,0.7)`

**Figure 10.12** RandBinomial Example Probability Mass Function**Figure 10.13** RandBinomial Example Cumulative Probability Function

#### RANDBINOMIAL Related Function

**FASTBINOMIAL:** Same as RANDBINOMIAL without any error checking of the arguments.

**CRITBINOM(trials,probability\_s,RAND()):** Excel's inverse of the cumulative binomial, or **CRITBINOM(trials,probability\_s,RANDUNIFORM(0,1))** to use the RiskSim Seed feature.

### RandBiVarNormal

Returns two random values from a bivariate normal distribution with specified means, standard deviations, and correlation.

To use this random number generator function, select two adjacent cells on the worksheet. Type **=RANDBIVARNORMAL** followed by numerical values for the five arguments or references to cells containing the values, separated by commas, enclosed in starting and ending parentheses.

After typing the ending parentheses, do not press Enter. Instead, hold down the Control and Shift keys while you press Enter, thus "array entering" the function.

Syntax:

`RANDBIVARNORMAL(mean1,stdev1,mean2,stdev2,correl12)`

Returns #REF! if the array function is not entered into two adjacent cells.

Returns #NUM! if a standard deviation is negative or the correlation is outside the range between -1 and +1.

Returns #VALUE! if an argument is not numeric.

Example: Select two adjacent cells, type

`=RANDBIVARNORMAL(100,10,50,5,0.5)`

Hold down Control and Shift while you press Enter.

RANDBIVARNORMAL Related Function

**FASTBIVARNORMAL:** Same as RANDBIVARNORMAL without any error checking of the arguments. The two adjacent cells must be in the same row.

## RandCumulative

Returns a random value from a piecewise-linear cumulative distribution. This function can model a continuous-valued uncertain quantity,  $X$ , by specifying points on its cumulative distribution. Each point is specified by a possible value,  $x$ , and a corresponding left-tail cumulative probability,  $P(X \leq x)$ . Random values are based on linear interpolation between the specified points.

RANDCUMULATIVE Syntax: `RANDCUMULATIVE(value_cumulative_table)`

Value\_cumulative\_table must be a reference, or the defined name of a reference, for a two-column range, with values in the left column and corresponding cumulative probabilities in the right column.

RANDCUMULATIVE Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if the first (top) cumulative probability is not zero, if the last (bottom) cumulative probability is not one, or if the values or cumulative probabilities are not in ascending order.

Returns #REF! if the number of columns in the table reference is not two.

Returns #VALUE! if the argument is not a reference, if the argument is a defined name but not for a reference, or if any cell of the table contains text or is blank.

**RANDCUMULATIVE Example** A corporate planner thinks that minimum possible market demand is 1000 units, median is 5000, and maximum possible is 9000. Also, there is a ten percent chance that demand will be less than 4000 and a ten percent chance it will exceed 7000. The values,  $x$ , and cumulative probabilities,  $P(X \leq x)$ , are entered into spreadsheet cells A2:B6.

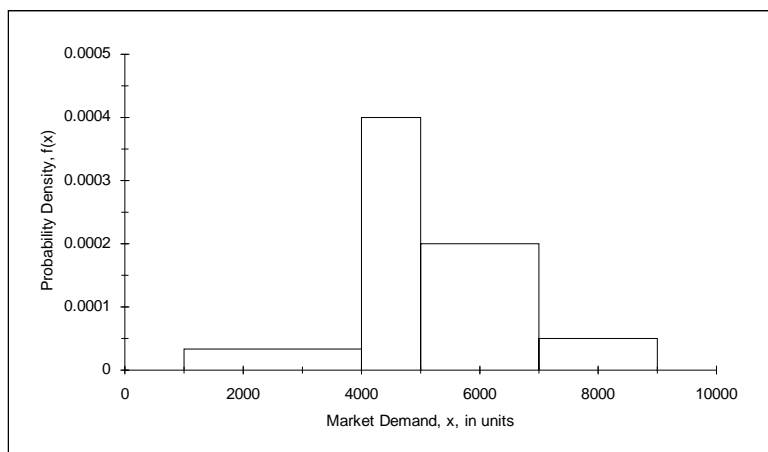
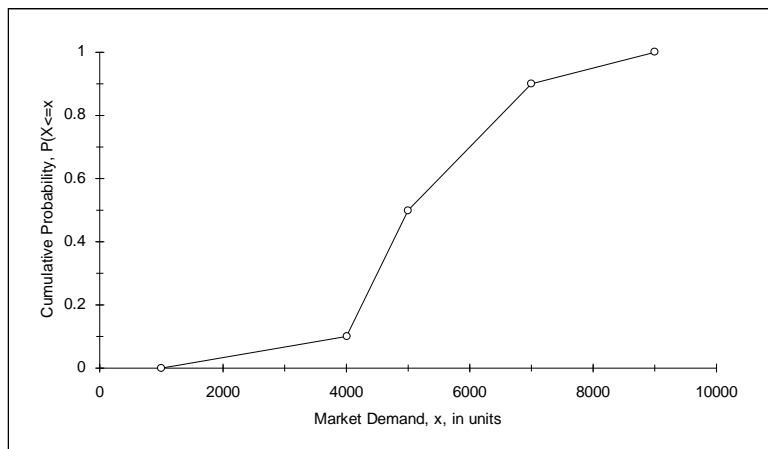
**Figure 10.14** RandDiscrete Example Spreadsheet Data

	A	B
1	x	$P(X \leq x)$
2	1000	0.0
3	4000	0.1
4	5000	0.5
5	7000	0.9
6	9000	1.0

The function is entered into another cell: =RANDCUMULATIVE(A2:B6)

RANDCUMULATIVE Related Function

FASTCUMULATIVE: Same as RANDCUMULATIVE without any error checking of the arguments.

**Figure 10.15** RandCumulative Example Probability Density Function**Figure 10.16** RandCumulative Example Cumulative Probability Function

## RandDiscrete

Returns a random value from a discrete probability distribution. This function can model a discrete-valued uncertain quantity,  $X$ , by specifying its probability mass function. The function is specified by each possible discrete value,  $x$ , and its corresponding probability,  $P(X=x)$ .

RANDDISCRETE Syntax: RANDDISCRETE(value\_discrete\_table)

Value\_discrete\_table must be a reference, or the defined name of a reference, for a two-column range, with values in the left column and corresponding probability mass in the right column.

### RANDDISCRETE Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

Returns #NUM! if a probability is negative or if the probabilities do not sum to one.

Returns #REF! if the number of columns in the table reference is not two.

Returns #VALUE! if the argument is not a reference, if the argument is a defined name but not for a reference, or if any cell of the table contains text or is blank.

### RANDDISCRETE Example

A corporate planner thinks that uncertain market revenue,  $X$ , can be approximated by three possible values and their associated probabilities:  $P(X=10000) = 0.25$ ,  $P(X=12000) = 0.50$ , and  $P(X=15000) = 0.25$ . The values and probabilities are entered into spreadsheet cells A2:B4.

**Figure 10.17** RandDiscrete Example Spreadsheet Data

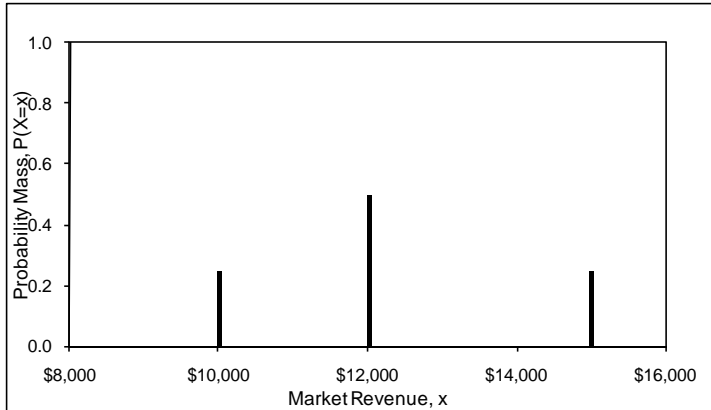
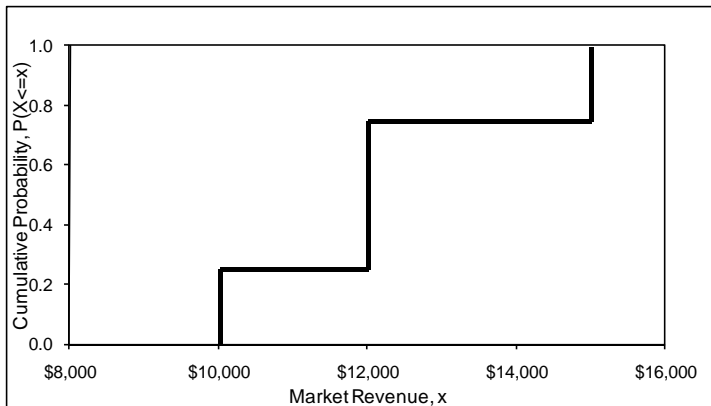
	A	B
1	$x$	$P(X=x)$
2	\$10,000	0.25
3	\$12,000	0.50
4	\$15,000	0.25

The function is entered into another cell: =RANDDISCRETE(A2:B4)

### RANDDISCRETE Related Function

FASTDISCRETE: Same as RANDDISCRETE without any error checking of the arguments.

RandDiscrete Example Probability Mass Function

**Figure 10.18** RandDiscrete Example Probability Mass Function**Figure 10.19** RandDiscrete Example Cumulative Probability Function

## RandExponential

Returns a random value from an exponential distribution. This function can model the uncertain time interval between successive arrivals at a queuing system or the uncertain time required to serve a customer.

**RANDEXPONENTIAL Syntax:** RANDEXPONENTIAL(lambda)

Lambda is the mean number of occurrences per unit of time.

**RANDEXPONENTIAL Remarks**

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

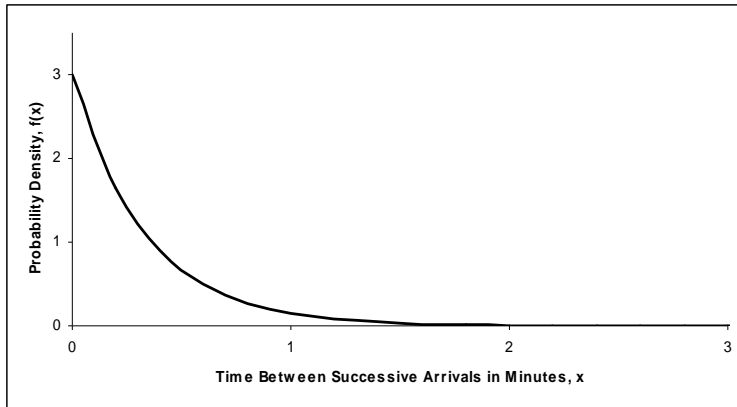
Returns #NUM! if lambda is negative or zero.

Returns #VALUE! if the argument is a defined name of a cell and the cell is blank or contains text.

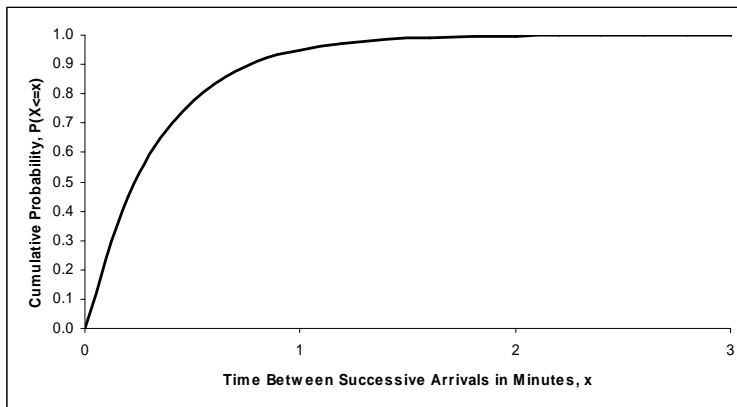
**RANDEXPONENTIAL Examples**

Cars arrive at a toll plaza with a mean rate of 3 cars per minute. The uncertain time between successive arrivals, measured in minutes, is =RANDEXPONENTIAL(3). The average value returned by repeated recalculation of RANDEXPONENTIAL(3) is 0.333.

**Figure 10.20** RandExponential Example Probability Density Function



**Figure 10.21** RandExponential Example Cumulative Probability Function



A bank teller requires an average of two minutes to serve a customer. The uncertain customer service time, measured in minutes, is =RANDEXPONENTIAL(0.5). The average value returned by repeated recalculation of RANDEXPONENTIAL(0.5) is 2.

#### RANDEXPONENTIAL Related Functions

**FASTEXPONENTIAL:** Same as RANDEXPONENTIAL without any error checking of the arguments.

–LN(RAND())/lambda: Excel's inverse of the exponential, or  
–LN(RANDUNIFORM(0,1))/lambda to use the RiskSim Seed feature.

**RANDPOISSON:** Counts number of occurrences for a Poisson process.

## **RandInteger**

Returns a uniformly distributed random integer between two integers you specify.

RANDINTEGER Syntax: RANDINTEGER(bottom,top)

Bottom is the smallest integer RANDINTEGER will return.

Top is the largest integer RANDINTEGER will return.

RANDINTEGER Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

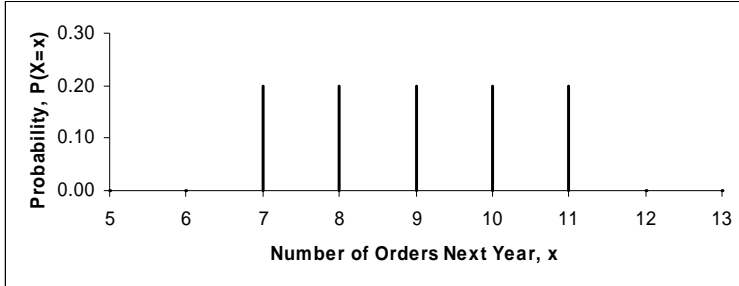
Returns #NUM! if top is less than or equal to bottom.

Returns #VALUE! if bottom or top is not an integer or if an argument is a defined name of a cell and the cell is blank or contains text.

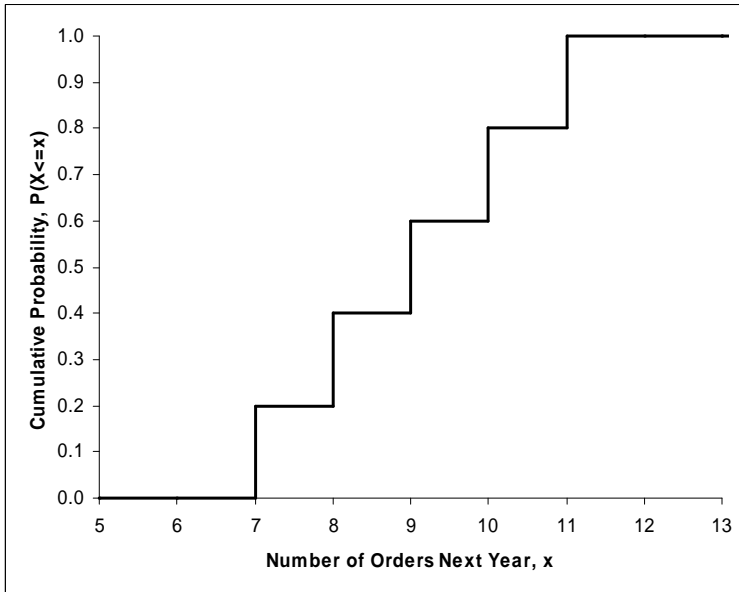
RANDINTEGER Example

The number of orders a particular customer will place next year is between 7 and 11, with no number more likely than the others. The uncertain number of orders is =RANDINTEGER(7,11).

**Figure 10.22** RandInteger Example Probability Mass Function



**Figure 10.23** RandInteger Example CumulativeProbability Function



**RANDINTEGER Related Function**

**FASTINTEGER:** Same as RANDINTEGER without any error checking of the arguments.

**RANDBETWEEN(bottom,top):** Excel's function for uniformly distributed integers, without RiskSim's capability of setting the seed.

**RandLogNormal**

Returns a random value from a lognormal distribution.

**RANDLOGNORMAL(mean,standard\_dev)**

## **RandNormal**

Returns a random value from a normal distribution. This function can model a variety of phenomena where the values follow the familiar bell-shaped curve, and it has wide application in statistical quality control and statistical sampling.

**RANDNORMAL Syntax:** RANDNORMAL(mean,standard\_dev)

Mean is the arithmetic mean of the normal distribution.

Standard\_dev is the standard deviation of the normal distribution.

**RANDNORMAL Remarks**

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

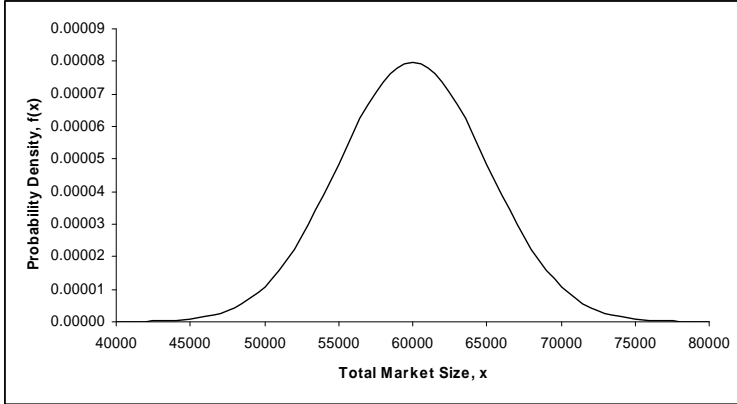
Returns #NUM! if standard\_dev is negative.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

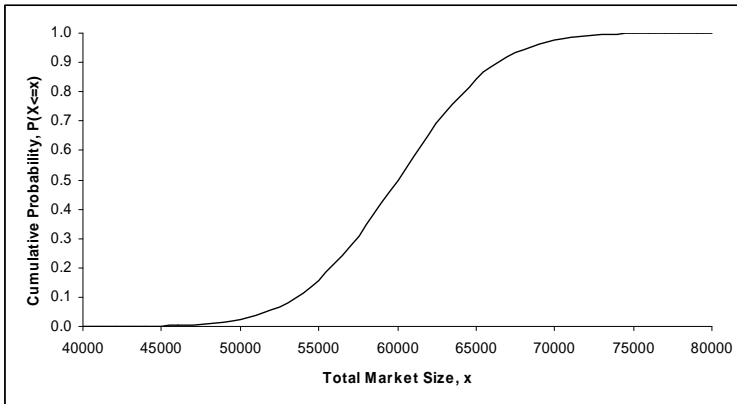
**RANDNORMAL Example**

The total market for a product is approximately normally distributed with mean 60,000 units and standard deviation 5,000 units. The uncertain total market is =RANDNORMAL(60000,5000).

**Figure 10.24** RandNormal Example Probability Density Function



**Figure 10.25** RandNormal Example Cumulative Probability Function



**RANDNORMAL Related Function**

**FASTNORMAL:** Same as RANDNORMAL without any error checking of the arguments.

**NORMINV(RAND(),mean,standard\_dev):** Excel's inverse of the normal, or **NORMINV(RANDUNIFORM(0,1),mean,standard\_dev)** to use the RiskSim Seed feature.

**RandPoisson**

Returns a random value from a Poisson distribution. This function can model the uncertain number of occurrences during a specified time interval, for example, the number of arrivals at a service facility during an hour. The possible values of RANDPOISSON are the non-negative integers, 0, 1, 2, ... .

**RANDPOISSON Syntax:** RANDPOISSON(mean)

Mean is the mean number of occurrences per unit of time.

**RANDPOISSON Remarks**

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if the argument is text and the name is undefined.

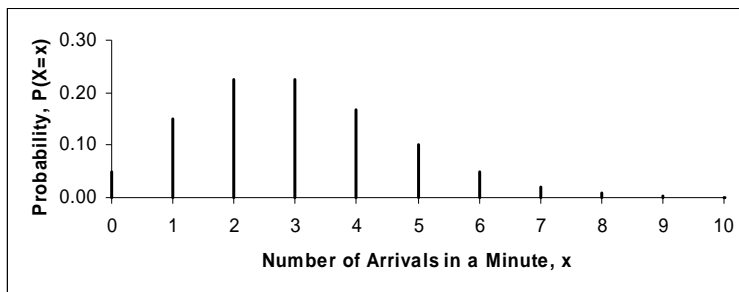
Returns #NUM! if mean is negative or zero.

Returns #VALUE! if mean is a defined name of a cell and the cell is blank or contains text.

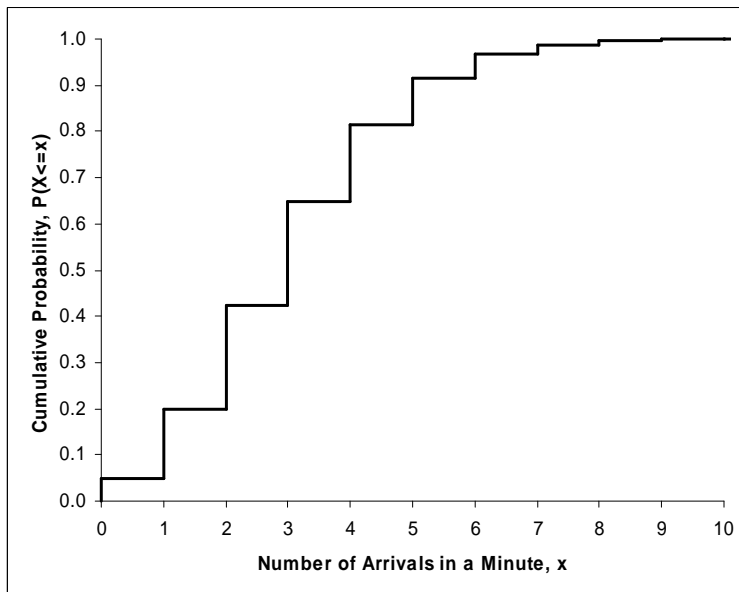
#### RANDPOISSON Examples

Cars arrive at a toll plaza with a mean rate of 3 cars per minute. The uncertain number of arrivals in a minute is =RANDPOISSON(3). The average value returned by repeated recalculation of RANDPOISSON(3) is 3.

**Figure 10.26** RandPoisson Example Probability Mass Function



**Figure 10.27** RandPoisson Example CumulativeProbability Function



A bank teller requires an average of two minutes to serve a customer. The uncertain number of customers served in a minute is =RANDPOISSON(0.5). The average value returned by repeated recalculation of RANDPOISSON(0.5) is 0.5.

### RANDPOISSON Related Functions

**FASTPOISSON:** Same as RANDPOISSON without any error checking of the arguments.

**RANDEXPONENTIAL:** Describes time between occurrences for a Poisson process.

## RandSample

Returns a random sample without replacement from a population.

To use this random number generator function, select a number of cells equal to the sample size, either in a single column or in a single row. Type =RANDSAMPLE followed by a reference to the cells containing the population values, enclosed in parentheses. After typing the ending parentheses, do not press Enter. Instead, hold down the Control and Shift keys while you press Enter, thus "array entering" the function.

Syntax: RANDSAMPLE(population)

The population argument is a reference to a range of values in a single column.

Returns #N/A if the population range is not part of a single column.

Returns #REF! if the function is not entered into two adjacent cells.

Example: Type population values into cells A2:A6. For a sample of size 3, select cells B2:B4, and type =RANDSAMPLE(A2:A6) but don't press Enter. Hold down Control and Shift while you press Enter. Press F9 to recalculate for a different sample.

	A	B
1	Population Data	Sample
2	29	73
3	73	57
4	13	29
5	44	
6	57	

**FastColumnSample:** Same as RandSample for two adjacent cells in the same column, without any error checking of arguments.

**FastRowSample:** Same as RandSample for two adjacent cells in the same row, without any error checking of arguments.

## RandTriangular

Returns a random value from a triangular probability density function. This function can model an uncertain quantity where the most likely value (mode) has the largest probability of occurrence, the minimum and maximum possible values have essentially zero probability of occurrence, and the probability density function is linear between the minimum and the mode and between the mode and the maximum. This function can also model a ramp density function where the minimum equals the mode or the mode equals the maximum.

RANDTRIANGULAR Syntax: RANDTRIANGULAR(minimum,most\_likely,maximum)

Minimum is the smallest value RANDTRIANGULAR will return.

Most\_likely is the most likely value RANDTRIANGULAR will return.

Maximum is the largest value RANDTRIANGULAR will return.

#### RANDTRIANGULAR Remarks

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

Returns #NUM! if minimum is greater than or equal to maximum, if most\_likely is less than minimum, or if most\_likely is greater than maximum.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

#### RANDTRIANGULAR Example

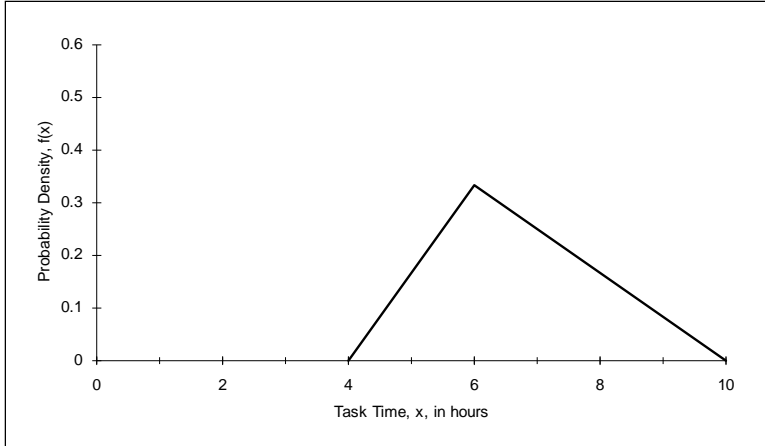
The minimum time required to complete a particular task that is part of a large project is 4 hours, the most likely time required is 6 hours, and the maximum time required is 10 hours.

The function returning the uncertain time required for the task is entered into a cell:  
=RANDTRIANGULAR(4,6,10).

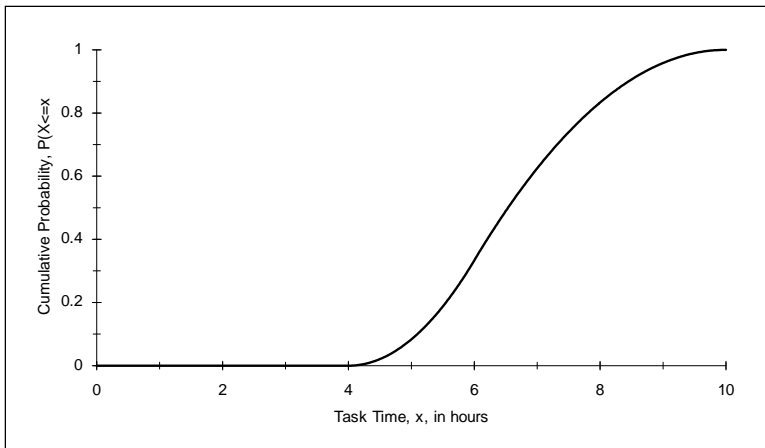
#### RANDTRIANGULAR Related Function

FASTTRIANGULAR: Same as RANDTRIANGULAR without any error checking of arguments.

**Figure 10.28** RandTriangular Example Probability Density Function



**Figure 10.29** RandTriangular Example Cumulative Probability Function



### RandTruncBiVarNormal

Returns a pair of random values from a bivariate normal probability density function with specified correlation, where each marginal distribution is truncated.

`RANDTRUNCBIVARNORMAL(mean1,stdev1,mean2,stdev2,correl12,min1,max1,min2,max2)`

### RandTruncNormal

Returns a random value from a truncated normal probability density function. This function can model an uncertain quantity with a bell-shaped density function where extreme values in the tails of the distribution are not desired. Subranges of the truncated UQ have the same relative probabilities as the same subranges of the original normal distribution.

`RANDTRUNCNORMAL`

Syntax: `RANDTRUNCNORMAL(Mean,StDev,MinValue,MaxValue)`

Mean and StDev are the parameters for the original normal distribution.

MinValue is the smallest value RANDTRUNCNORMAL will return.

MaxValue is the largest value RANDTRUNCNORMAL will return.

MinValue and MaxValue are optional arguments.

**RANDTRUNCNORMAL** Remarks

Returns #N/A if a value between MinValue and MaxValue is not found after 10,000 attempts.

Returns #NUM! if MinValue is greater than or equal to MaxValue.

Returns #VALUE! if an argument cannot be interpreted as a numeric value.

**RANDTRUNCNORMAL** Example

An uncertain quantity has a normal distribution with mean = 100 and standard deviation = 10, but values are restricted between 90 and 120.

The function returning the uncertain quantity is entered into a cell: = RANDTRUNCNORMAL (100,10,90,120).

**RANDTRUNCNORMAL** Related Function

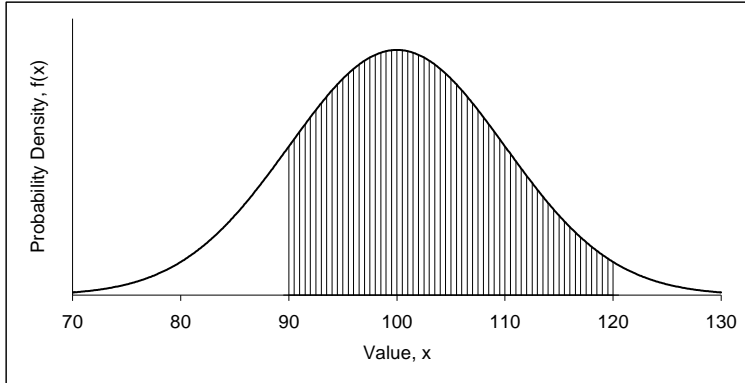
**FASTTRUNCNORMAL**: Same as RANDTRUNCNORMAL, but with error checking only for maximum of 10,000 attempts.

Figure 10.30 shows the original normal density function with mean = 100 and standard deviation = 10. The density function is rescaled so that the total area under the curve between 90 and 120 is equal to one. The corresponding cumulative distributions are shown in Figure 10.31.

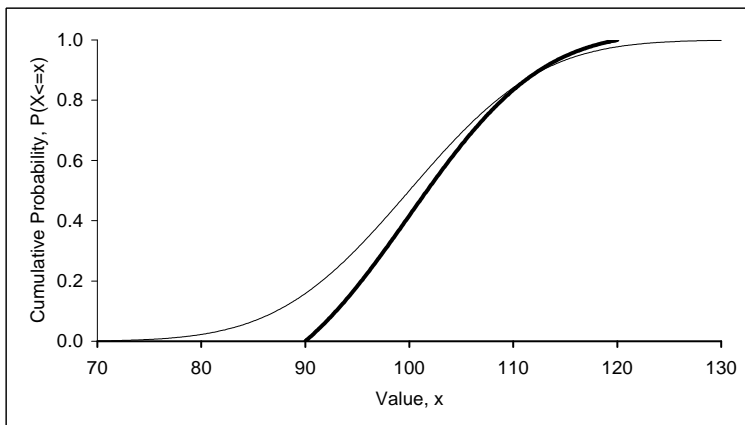
For example, with the original normal density function,  $P(100 \leq X \leq 110) = 0.341$ , and  $P(110 \leq X \leq 120) = 0.136$ . The ratio of  $P(100 \leq X \leq 110)$  to  $P(110 \leq X \leq 120)$  is  $0.341/0.136 = 2.512$ . Thus, a value in the range 100 to 110 is approximately 2.5 times as likely as a value in the range 110 to 120.

With the truncated normal density function,  $P(100 \leq X \leq 110) = 0.417$ , and  $P(110 \leq X \leq 120) = 0.166$ . The ratio of  $P(100 \leq X \leq 110)$  to  $P(110 \leq X \leq 120)$  is  $0.417/0.166 = 2.512$ . Thus, a value in the range 100 to 110 is still approximately 2.5 times as likely as a value in the range 110 to 120.

**Figure 10.30** RandTruncNormal Probability Density Function



**Figure 10.31** RandTruncNormal Cumulative Probability Function



## RandUniform

Returns a uniformly distributed random value between two values you specify. As a special case, RANDUNIFORM(0,1) is the same as Excel's built-in RAND() function.

**RANDUNIFORM Syntax:** RANDUNIFORM(minimum,maximum)

Minimum is the smallest value RANDUNIFORM will return.

Maximum is the largest value RANDUNIFORM will return.

**RANDUNIFORM Remarks**

Returns #N/A if there are too few or too many arguments.

Returns #NAME! if an argument is text and the name is undefined.

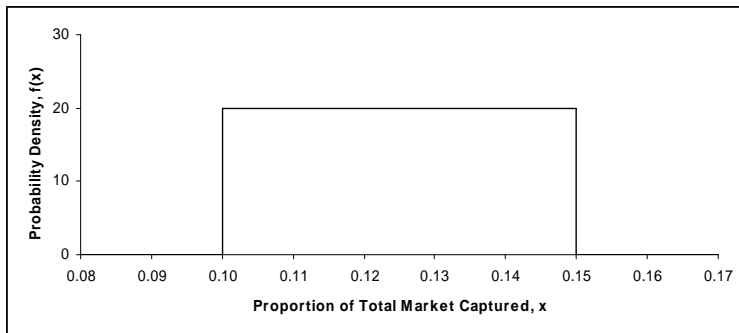
Returns #NUM! if minimum is greater than or equal to maximum.

Returns #VALUE! if an argument is a defined name of a cell and the cell is blank or contains text.

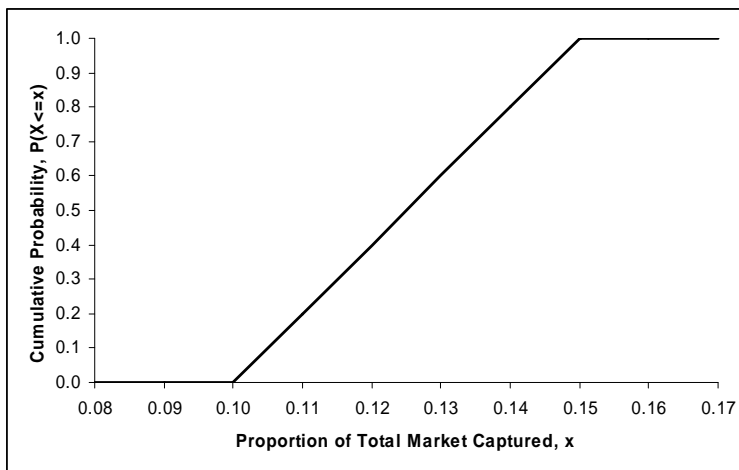
**RANDUNIFORM Example**

A corporate planner thinks that the company's product will garner between 10% and 15% of the total market, with all possible percentages equally likely in the specified range. The uncertain market proportion is =RANDUNIFORM(0.10,0.15).

**Figure 10.32** RandUniform Example Probability Density Function



**Figure 10.33** RandUniform Example Cumulative Probability Function



#### RANDUNIFORM Related Function

**FASTUNIFORM:** Same as RANDUNIFORM without any error checking of the arguments.

## 10.10 RISKSIM TECHNICAL DETAILS

RiskSim's random number generator functions are based on a uniformly distributed random number function called RandSeed which is not directly accessible by the user. RandSeed returns a random value  $x$  in the range  $0 < x \leq 1$ . Internally, decimal values for RandSeed are calculated by dividing a uniformly distributed random integer by 2,147,483,647, which is RandSeed's period. Random integers in the range 1 through 2,147,483,647 are generated using the well-documented Park-Miller algorithm, where each random integer depends on the previous random integer.

When RiskSim starts, the initial integer seed depends on the system clock. Unlike Excel's RAND() function, you can use RiskSim at any time to specify an integer seed in the range 1 through 2,147,483,647, which is used as the previous random integer for the sequence of random numbers generated by the RiskSim functions.

In the Risk Simulation dialog box, the "Random number seed" edit box changes the seed only for the RiskSim functions; it does not have any effect on Excel's built-in RAND() function.

Each of RiskSim's random number generator functions use RandSeed as a building block.

RANDBINOMIAL(trials,probability\_s) uses RandSeed as the cumulative probability in Excel's built-in CRITBINOM function.

RANDBIVARNORMAL(mean1,stdev1,mean2,stdev2,correl12) uses two values of RandNormal to obtain correlated normal values.

RANDCUMULATIVE(value\_cumulative\_table) uses the value of RandSeed, R, searches to find the adjacent cumulative probabilities that bracket R, and interpolates on the linear segment of the cumulative distribution to find the corresponding value.

RANDDISCRETE(value\_discrete\_table) compares RandSeed with summed probabilities of the input table until the sum exceeds the RandSeed value, and then returns the previous value from the input table.

RANDEXPONENTIAL(lambda) uses the value of RandSeed, R, as follows. If the exponential density function is  $f(t) = \lambda \cdot \text{EXP}(-\lambda \cdot t)$ , the cumulative is  $P(T \leq t) = 1 - \text{EXP}(-\lambda \cdot t)$ . Associating R with  $P(T \leq t)$ , the inverse cumulative is  $t = -\text{LN}(1-R)/\lambda$ . Since R and 1-R are both uniformly distributed between 0 and 1, RiskSim uses  $-\text{LN}(R)/\lambda$  for the returned value.

RANDINTEGER(bottom,top) returns  $\text{bottom} + \text{INT}(\text{RandSeed} \cdot (\text{top} - \text{bottom} + 1))$ .

RANDNORMAL(mean,standard\_dev) uses two RandSeed values in the well-documented Box-Muller method.

RANDPOISSON(mean) compares RandSeed with cumulative probabilities of Excel's built-in POISSON function until the probability exceeds the RandSeed value, and then returns the previous value.

RANDSAMPLE(population) uses RandSeed for each of the cells that were selected when the function was array-entered, avoiding population values that have already been selected, thus providing sampling without replacement.

RANDTRIANGULAR(minimum,most\_likely,maximum) uses RandSeed once. The triangular density function has two linear segments, so the cumulative distribution has two quadratic segments. The returned value is determined by interpolation on the appropriate quadratic segment.

RANDTRUNCNORMAL(Mean,StDev,MinValue,MaxValue) uses values of RandNormal until a value is found between MinValue and MaxValue or until it has made 10,000 attempts.

RANDUNIFORM(minimum,maximum) returns  $\text{minimum} + \text{RandSeed} \cdot (\text{maximum} - \text{minimum})$ .  
 RANDUNIFORM(0,1) is equivalent to Excel's built-in RAND() function.

RiskSim includes a FAST... version of each of the fourteen functions, e.g., FASTBINOMIAL, FASTCUMULATIVE, etc. The FAST... functions are identical to the RAND... functions except there is no error checking of arguments.