

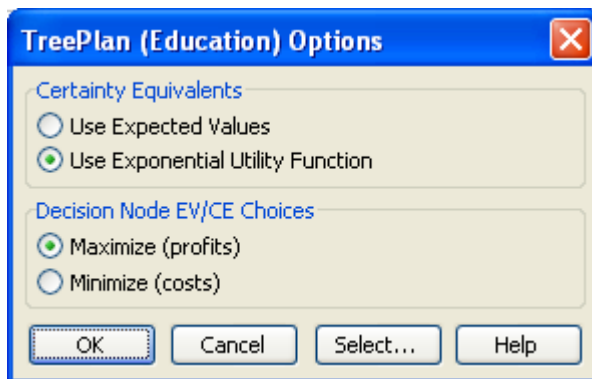
# Risk Attitude Using TreePlan

# 22

## 22.1 EXPONENTIAL UTILITY AND TREEPLAN

TreePlan's default is to rollback the tree using expected values. If you choose to use exponential utilities in TreePlan's Options dialog box, TreePlan will redraw the decision tree diagram with formulas for computing the utility and certain equivalent at each node.

Figure 22.1 TreePlan Options Dialog Box



For the Maximize option, the rollback formulas are  $U = A - B * \text{EXP}(-X/RT)$  and  $CE = -\text{LN}((A - EU)/B) * RT$ , where X and EU are cell references. For the Minimize option, the formulas are  $U = A - B * \text{EXP}(X/RT)$  and  $CE = \text{LN}((A - EU)/B) * RT$ .

TreePlan uses the name RT to represent the risk tolerance parameter of the exponential utility function. The names A and B determine scaling. If the names A, B, and RT don't exist on the worksheet when you choose to use exponential utility, they are initially defined as A=1, B=1, and RT=999999999999. You can redefine the names using the Insert | Name | Define or Insert | Name | Create commands.

To plot the utility curve, enter a list of X values in a column on the left, and enter the formula  $=A - B * \text{EXP}(-X/RT)$  in a column on the right, where X is a reference to the corresponding cell on the left. Select the values in both columns, and use the ChartWizard to develop an XY (Scatter) chart.

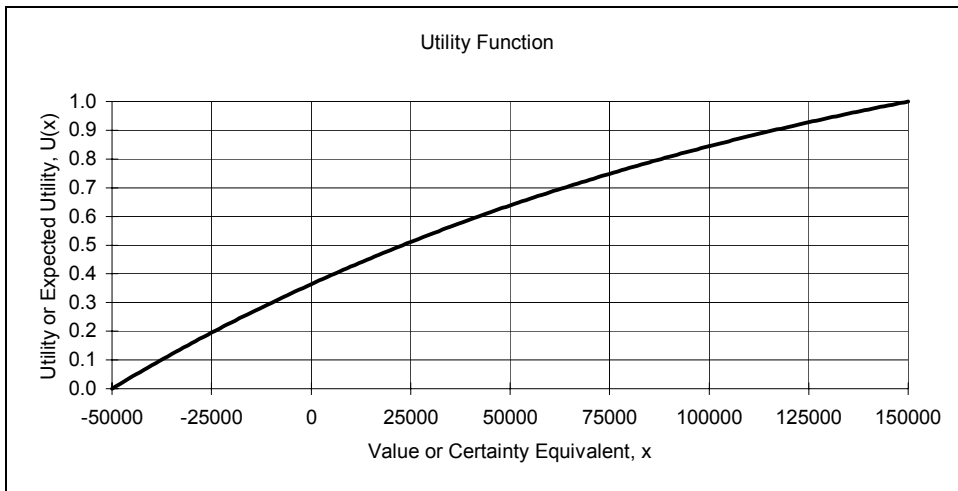
If RT is specified using approximate risk tolerance values, you can perform sensitivity analysis by (1) using the defined name RT for a cell, (2) constructing a data table with a list of possible RT values and an appropriate output formula (usually a choice indicator at a decision node or a certain equivalent), and (3) specifying the RT cell as the input cell in the Data Table dialog box.

## 22.2 RISK UTILITY SUMMARY

### Concepts

Strategy, Payoff Distribution, Certain Equivalent

**Figure 22.2** Utility Function



### Fundamental Property of Utility Function

The utility of the CE of a lottery equals the expected utility of the lottery's payoffs.

$$U(\text{CE}) = \text{EU} = p_1 \cdot U(x_1) + p_2 \cdot U(x_2) + p_3 \cdot U(x_3)$$

### Using a Utility Function To Find the CE of a Lottery

1.  $U(x)$ : Locate each payoff on the horizontal axis and determine the corresponding utility on the vertical axis.
2.  $\text{EU}$ : Compute the expected utility of the lottery by multiplying each utility by its probability and summing the products.
3.  $\text{CE}$ : Locate the expected utility on the vertical axis and determine the corresponding certain equivalent on the horizontal axis.

### Exponential Utility Function

General form:  $U(x) = A - B \cdot \text{EXP}(-x/\text{RT})$

Parameters A and B affect scaling.

Parameter RT (RiskTolerance) depends on risk attitude and affects curvature.

$$\text{Inverse: CE} = -\text{RT} \cdot \text{LN}[(A - \text{EU})/B]$$

### TreePlan's Simple Form of Exponential Utility

Set A and B equal to 1.

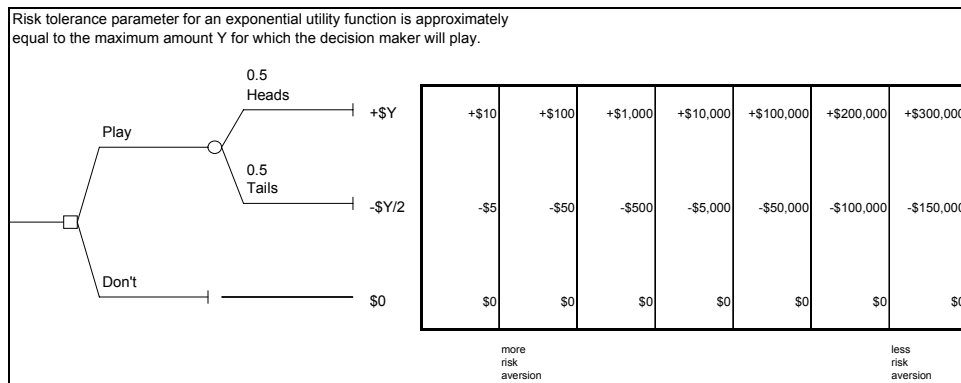
$$U(x) = 1 - \text{EXP}(-x/RT)$$

$$CE = -RT * \text{LN}(1 - EU)$$

### Approximate Assessment of RiskTolerance

Refer to the Clemen textbook, Figure 13.12, on page 478.

Figure 22.3 Assessing Approximate Risk Tolerance



### Exact Assessment of RiskTolerance

The RISKTOL.XLA Excel add-in file adds the following function to the Math & Trig function category list:

$$\text{RISKTOL}(\text{WorsePayoff}, \text{CertEquiv}, \text{BetterPayoff}, \text{BetterProb})$$

The first three arguments are required, and the last argument is optional with default value 0.5. WorsePayoff and BetterPayoff are payoffs of an assessment lottery, and CertEquiv is the decision maker's certain equivalent for the lottery.

RISKTOL returns #N/A if there are too few or too many arguments, #VALUE! if WorsePayoff >= CertEquiv, or CertEquiv >= Better Payoff, or BetterProb (if specified) <= 0 or >= 1, and #NUM! if the search procedure fails to converge.

For example, consider a 50-50 lottery with payoffs of \$100,000 and \$0. A decision maker has decided that the certain equivalent is \$43,000. If you open the RISKTOL.XLA file and type =RISKTOL(0,43000,100000) in a cell, the result is 176226. Thus, the value of the RiskTolerance parameter in an exponential utility function for this decision maker should be 176226.

### Using Exponential Utility for TreePlan Rollback Values

1. Select a cell, and enter a value for the RiskTolerance parameter.
2. With the cell selected, choose Insert Name | Define, and enter RT.
3. From TreePlan's Options dialog box, select Use Exponential Utility. The new decision tree diagram includes the EXP and LN functions for determining U(x) and the inverse.

### **Using Exponential Utility for a Payoff Distribution**

Enter the exponential utility function directly, using the appropriate value for RiskTolerance. If the payoff values are equally-likely, use the AVERAGE function to determine the expected utility; otherwise, use SUMPRODUCT. Enter the inverse function directly to obtain the certain equivalent.